

Optimal Social Insurance and Health Inequality*

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Abstract

This paper integrates into public economics a biologically founded, stochastic process of individual ageing. The novel approach enables us to quantitatively characterize the optimal joint design of health and retirement policy behind the veil of ignorance for today and in response to future medical progress. Calibrating our model to Germany, our analysis suggests that the current social insurance policy instruments are set close to the (constrained) socially optimal levels, given proportional contribution rates for health and pension finance, the equivalence principle in the pension system, and a common statutory retirement age. Future progress in medical technology calls for a potentially drastic increase in health spending and a higher retirement age without lowering the pension contribution rate. Interestingly, from an *ex ante* point of view, medical progress and higher health spending are in conflict with the goal to reduce health inequality.

Key words: Ageing; Health Expenditure; Health Inequality; Social Security System; Retirement Age.

JEL classification: H50; I10; C60.

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1 Introduction

Life expectancy of adults increased by around 15 years over the 20th century and many researchers in demography and the natural sciences consider it likely to increase further (e.g. Oeppen and Vaupel, 2002). The development was to a large extent driven by fast advances in medical and pharmaceutical research that led to substantial increases in the effectiveness of health spending on the ageing process (e.g. Lichtenberg, 2007).¹ Through this channel, health spending contributes to the widely discussed problem of social security systems, namely that pension benefits are likely to decline for a given pension savings rate unless the statutory retirement age increases. This implies that health and pension policy shall be examined and designed jointly.

Higher effectiveness of medical technology may make higher contribution rates to public health insurance more desirable in order to better reap the benefits in terms of improved health and higher life expectancy. Its adverse effect on pension benefits could be offset by raising contributions to the pension system as well. There is, however, a trade-off between the two tiers of the social insurance system, as higher contributions to pension insurance and health insurance both reduce net income in the working period, for given labor supply and because of a reduction in labor supply. Alternatively, to limit tax distortions, it could be desirable to increase the retirement age along with an increase in health spending. In view of the complex linkages between the pension system and the health care system created by the endogeneity of human health and longevity, the jointly optimal design of our social insurance systems appears to be both important and *a priori* non-obvious.

This paper develops a calibrated model for Germany to examine the interactions between public health and pension policy in order to quantitatively characterize the optimal joint design of the social insurance system today and in response to future medical progress. In line with a long tradition in public economics to justify social systems, we focus on expected welfare maximization at the beginning of life (e.g. Sinn, 1995). Specifically, we investigate to

¹Medical and pharmaceutical innovations became important drivers of life expectancy from the 1950s onwards. Before that, life expectancy rose predominantly because of reductions in child mortality (e.g. Milligan and Wise, 2011). For instance, in the U.S., the fraction of the population which is at least 65 years old is projected to be 18.8 percent in 2025, whereas it was 8.1 percent in 1950 (Poterba, 2014).

which degree an *ex ante* identical population would agree, behind the veil of ignorance, on a social insurance system which reduces the probability of illness and insures against social hardships and long life. For practical reasons, we confine ourselves to derive the constrained optimal policy mix, given the proportional contribution rates to public health and pension finance, the equivalence principle in the pension system in Germany, and the existence of a common statutory retirement age.

An interesting, related question is whether implementing an *ex ante* optimal health care system will reduce health inequality within a society, compared to the status quo. In fact, reducing health inequality is a major goal of large organizations like the World Health Organization (WHO) and the European Union (EU).² It is, however, non-obvious whether it is in line or in conflict with the goal to maximize *ex ante* welfare.³

Our key innovation which enables us to examine these issues is to integrate into public economics a biologically founded process of individual ageing, based on empirical evidence from gerontology. Ageing is understood as the stochastic and individual-specific deterioration of the functioning of body and mind – represented by an accumulation of health deficits – ranging from mild deficits like reduced vision or incontinence to near lethal ones like stroke. Our framework captures the features that (i) at any given age, the number of health deficits is approximately Poisson-distributed in the population, (ii) the average number of individual health deficits grows with age, and (iii) the probability of death at any age strongly depends on the number of health deficits an individual has accumulated over time (Mitnitski and Rockwood, 2002a, 2002b, 2005; Masaro, 2006). Thus, in a stochastic sense, the number of health deficits as retiree depends positively on the number of health deficits in working age (Mitnitski et al., 2006, 2007a, 2007b). The health deficit approach enables us to calibrate the health technology based on a health status measure that is easily observable and empirically

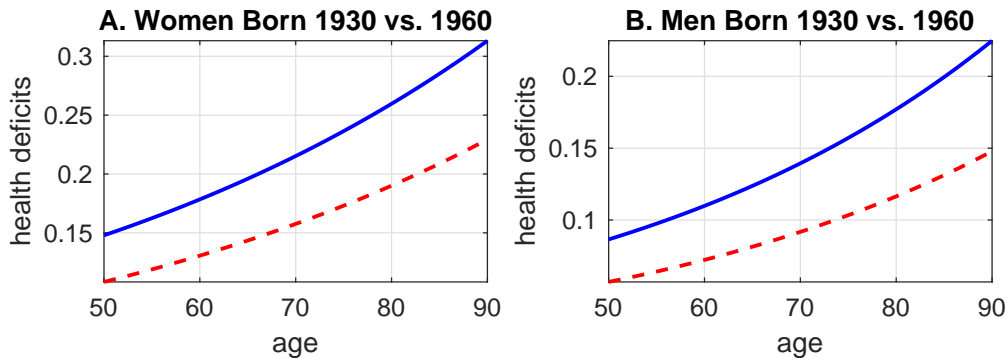
²See www.who.int and www.health-inequalities.eu/. According to the WHO, health inequality is defined as “differences in health status or in the distribution of health determinants between different population groups”. Chetty et al. (2016) documents health inequality in the US. Similar socioeconomic gradients are observable in Europe, including Germany, albeit at a somewhat lower level (Kunst et al., 2005; Harttgen et al., 2013).

³*Ex post*, by contrast, a social planner with a redistributive motive may well want to mitigate health inequality after risk has materialized. This is, however, not our focus in this paper.

successful in predicting life expectancy.⁴

Improvements of health status over time can be documented by looking at the evolution of the number of health deficits as a fraction of potential deficits for a given age – the so-called *health deficit index*. Figure 1 illustrates how health deficits are accumulated with age in Germany by women and men. It shows the predicted average health deficits of cohorts born in 1930 (solid lines) and 1960 (dashed lines). The predictions are taken from the estimates in Abeliansky and Strulik (2018) who compute a health deficit index for a panel of 14 European Countries and six waves of the Survey of Health, Aging, and Retirement in Europe (SHARE).⁵ As individuals age chronologically, they develop more health deficits at a rate of, on average, about 2 percent per year. Men are initially less frail but age slightly faster than women. The global improvement in human health, understood as progress in the delay of human aging, is shown by the downward shift of the health deficit trajectories. For example, the level of health deficits experienced at age 55 by individuals born 1930 is predicted to be experienced at age 70 by individuals born 1960.

Figure 1. Health Deficit Accumulation of Women and Men in Germany



The figure shows the average health deficit index by age. Panel A: solid (blue) line: Women born 1930, dashed (red) line: women born 1960. Panel B: solid (blue) line: Men born 1930, dashed (red) line: men born 1960. Estimates from Abeliansky and Strulik (2018; Tab. 3, Tab. 4).

We focus on a simple framework with a working period and a retirement period where, in

⁴Notwithstanding the advances in the natural sciences to understand life cycle health, the common conceptualization of health in economics is still based on the Grossman (1972) model. The basic idea in that approach is that individuals accumulate health through investment in health capital, a latent variable.

⁵Abeliansky and Strulik (2018) estimate the cohort-specific development of health deficits according to the methodology developed by Mitnitski and Rockwood (2002a, 2002b). They show that for each year of later birth, health deficits decline by an almost constant trend of 1.4-1.5 percent and argue that this trend approximates the rate of medical progress, broadly defined. In Appendix A we provide details of the estimation method and the compilation of the health deficit index.

contrast to standard overlapping generations models, longevity (i.e. the length of the retirement period) is stochastic and endogenous to the generosity of the health care system. We distinguish health care expenses for the working-aged from health expenditure targeted to typical illnesses of the elderly. Because of the stochastic path-dependency in the evolution of individual health deficits, health expenditures targeted to the working-aged affect the distribution of health deficits also among retirees. Consequently, improving health of the working-aged raises life expectancy for individuals at retirement age and *ceteris paribus* reduces pension benefits. It also raises labor supply and therefore generates more contributions to the social insurance system, with positive effects on pension benefits.

An important assumption that keeps the analysis tractable and the numerical results well interpretable is that workers fully rely on the public pay-as-you-go (PAYG) pension system to finance old-age consumption. Indeed, private old age savings quantitatively play a minor role in Germany.⁶ We also focus on a public health insurance system. In Germany, only top earners, civil servants and self-employed may opt out of the public PAYG health system.⁷

The main results from our numerical analysis may be summarized as follows. First, the *status quo* health system in Germany is close to the constrained social optimum. The pension savings rate should, if anything, moderately increase from its *status quo* level whereas the desired and actual statutory retirement age deviate little from each other. Second, the possibility to prolong life via *future* medical progress shall be exploited by increasing the health contribution rate – and drastically so in some cases. Third, whereas the optimal pension contribution rate should increase by little in the future, the optimal future retirement age should typically be higher than today. The optimal retirement age, however, increases by less than life expectancy in most of our analyzed cases. Fourth, more health spending as an optimal response to a more powerful medical technology leads to more relative inequality of health (as, for instance, measured by the Gini coefficient of health deficits) at old age. In other words, there are disproportionately large health gains for those who develop only a

⁶Retired households received about 80 percent of income from social security in the 1990s (Börsch-Supan and Schnabel, 1998). Empirical evidence also suggests that assuming agents who do not adjust private savings when public pension policy changes well describes the behavior of the vast majority of individuals (Chetty, 2015).

⁷Consequently, 90 percent of employees in Germany were publicly health-insured in the year 2011 (Statistisches Bundesamt, 2015).

small number of health deficits.

There exists a relatively large literature discussing the impact of social security on labor supply and retirement and on the optimality or sustainability of public pension systems (e.g. Auerbach and Kotlikoff, 1987; Imrohoroglu et al., 1995; Börsch-Supan, 2000; Conesa and Krueger, 1999; Feldstein and Liebman, 2002; Conesa et al., 2009; Jaag et al., 2010, 2011). This literature, however, does not investigate the interaction of pension finance and health care.

A smaller literature examines the impact of health on labor supply and retirement (Wolfe, 1985; Philipson and Becker, 1998; French, 2005; Pestieau et al., 2008; Heijdra and Romp, 2009; French and Jones, 2011; Leroux et al., 2011a, 2011b; Imrohoroglu and Kital, 2012; Bloom et al., 2014; Kuhn and Prettnner, 2016). While the available studies point to various interesting interactions of health and public policy, they typically focus on standard two-period overlapping generations models where the risk lies in whether an individual reaches the second period. Thus, they abstract from the effect of health on longevity, i.e. the years spent in retirement. Moreover, the path-dependency of health in working age and health in old age, emphasized in the gerontology literature, remained unexplored. In our model, both the probability to reach the retirement age and the years spent in retirement are endogenous. This allows us to examine the role of health spending and its allocation over the life-cycle on pension income via the effects on longevity.

Possibly most related to our paper is the study by Hall and Jones (2007). Comparing the actual and socially optimal health spending in the U.S., their analysis suggests that current health spending in the U.S. is too low. The key difference of our model to their work is the modeling of ageing and the resulting effect of health expenditure on health status. Hall and Jones (2007) assume a functional relationship between contemporaneous health spending and contemporaneous health status (measured by the inverse of the age- and time-specific mortality rate). In contrast, we model ageing as an accumulation of health deficits. Thus, health spending in young age affects health status also when old via the path-dependency of health deficits – a channel that is absent in Hall and Jones (2007). Thus, the model calibrated by Hall and Jones (2007) implies potentially large effects of health spending at old age relative to that in young age. Given that the greater part of health spending

is concentrated at old age, this modeling strategy likely biases optimal health spending upwards.⁸ Our study, however, agrees with that of Hall and Jones (2007) by arguing that further improving medical progress calls for substantial increases of health spending in the future.

Finally, in economics, the health deficit approach has been almost exclusively employed to analyze life-cycle decisions of a single agent (Dalgaard and Strulik, 2014, 2017; Schuenemann et al., 2017a, 2017b; Strulik, 2018).⁹ In this paper, we examine economy-wide interactions of health spending and endogenous longevity with the pension system, and its implications for health inequality.

The paper is structured as follows. Section 2 presents the theoretical model, capturing the main features of the German public PAYG system for both health care and pensions. Section 3 calibrates our framework to Germany. Section 4 conducts numerical analysis to derive the constrained optimal joint design of health and pension policy behind the veil of ignorance. Section 5 examines how the optimal policy design should adjust when medical technology further improves and explores the implications on health, life expectancy, and health inequality. The last section concludes.

2 The Model

Consider the following continuous-time model. At each date t , a new cohort of *ex ante* identical individuals is born. The cohort size is time-invariant and normalized to unity. This assumption reflects our focus on the effects of ageing on the social insurance system caused by higher life expectancy rather than by (presumably temporary) changes in the birth rate.

Life consists of a working period and a retirement period. *Ex post* individuals differ regarding health status. The individual-specific deterioration of health is stochastic and

⁸Aside from focussing on Germany instead of the U.S., we also deviated from Hall and Jones (2008) by investigating the interaction between health and pension policy. These differences limit comparability between results.

⁹An exception is Böhm et al. (2017) who predict future longevity under different scenarios of health care access (i.e. rationing scenarios). This study neither considers a public pension system nor does it allow for within-cohort health inequality, in contrast to the present paper.

depends on the health care system. Both the probability to reach the retirement period and the individual length of the retirement period are endogenous to the health status.

2.1 The Social Insurance System

We aim to capture the main features of the German health and pension system. The maximum length of the working period is denoted by \bar{R} and it is the same for all individuals. It can be thought of the statutory retirement age. The government provides a health care system and a pension system to maximize *ex ante* welfare. Health expenditure and pension benefits are financed by proportional social insurance contributions. There are separate budgets with health care contribution rate $\tau_h \in (0, 1)$ on income of the working-aged and retirees and pension contribution rate $\tau_s \in (0, 1)$ on labor income. Both systems are pay-as-you-go (PAYG), i.e. the revenues are paid out contemporaneously and the budgets are balanced. We distinguish between health spending targeted to the working-aged population (e.g. for prevention programmes and curative care of illnesses that typically also hit younger adults, like virus infections and psychiatric problems) and health spending targeted to retirees (e.g. for treating illnesses typically related to old age, like cardiovascular diseases, cancer and arthrosis). The pension system reflects the equivalence principle: the relative size of contributions of individuals is proportional to the relative size of their pension benefits. Pension benefits are time-invariant for an individual during the retirement period.

In addition, the government taxes labor income progressively. For simplicity, we assume a co-linear tax schedule that reads as $\tau_w I - \mathcal{T}$, where I is labor income, τ_w is the marginal tax rate and \mathcal{T} is referred to as earned income tax credit. As will become apparent, individuals with lower health status will supply less labor. Assuming a balanced budget, labor income taxation is therefore redistributive.

We abstract from private forms of health expenditure and pension insurance. Specifically, a private annuity market is missing and individuals cannot save privately for the retirement period. This captures, albeit in a pronounced way, the insignificance of private savings for retirement wealth for the vast majority of households in Germany for which we calibrate

our model.¹⁰ Assuming non-optimizing households with respect to old-age consumption is consistent with evidence from behavioral economics showing that most individuals stick to default pension plans offered by their employers (e.g. Chetty, 2015). Such evidence widely opens the scope for publicly financing social insurance, as discussed in Beshears et al. (2009).

2.2 Production

At each date, there is a single homogenous consumption good which is produced according to a neoclassical, constant-returns-to-scale production technology. Output Y is given by

$$Y = F(K, AL) \equiv ALf(k), \quad k \equiv \frac{K}{AL}, \quad (1)$$

where K and L are the inputs of physical capital and labor, the latter being measured in efficiency units. A is an exogenous measure of productivity. $f(\cdot)$ is strictly increasing, strictly concave, and fulfills the Inada conditions.

Output is sold in a perfectly competitive environment. The output price is normalized to unity. The rate of return to capital, r , is internationally given (i.e. we consider a small open economy assuming capital income is not taxed) and time-invariant. Thus, profit maximization of the representative firm implies that k is given by $r = f'(k)$, i.e. $k = (f')^{-1}(r) \equiv \bar{k}(r)$. Consequently, the wage rate per efficiency unit of human capital reads as $w = A\omega$ with $\omega \equiv f(\bar{k}(r)) - \bar{k}(r)f'(\bar{k}(r))$.

2.3 Individuals

The individual number of health deficits can assume values from the set $S = \{0, 1, \dots, \bar{n}\}$. Individuals are indexed by $i \in [0, 1]$. The number of health deficits of individual i in the

¹⁰For instance, in the year 2002, Germany introduced a subsidized private annuity market that was similar to the subsidized IRA accounts in the U.S. The public subsidies are especially high as a percentage of contributions for low income households with children. Until the end of March 2008, about 11 million contracts have been signed. Nevertheless, the impact of the subsidies on savings in terms of private annuities was negligible (Corneo et al., 2009; Börsch-Supan et al., 2015). This suggests that the saving volumes defined in the contracts are low and/or have replaced other forms of private annuity savings (which have anyway low volumes for the great majority of households).

working period (indexed by subscript “1”) and retirement period (indexed by “2”) are denoted by $n_1(i)$ and $n_2(i)$, respectively. An individual i reaches the retirement age if $n_1(i)$ is sufficiently low. As will become apparent, the distribution out of which an individual draws n_2 is conditional on the individual realization of n_1 .

Life-time is determined by a strictly decreasing function of health deficits, $\tilde{T}(n)$ for $n \in S$ (Mitnitski et al., 2005, 2007). Individual i reaches the retirement age if $\tilde{T}(n_1(i)) \geq \bar{R}$ and dies before age \bar{R} otherwise; in the latter case, life-time is given by $\tilde{T}(n_1(i))$. Let $\bar{S} \equiv \{n \in S : n > \tilde{T}^{-1}(\bar{R})\}$ denote the set of health deficits in working age for which an individual does not reach retirement age and $\underline{S} \equiv \{n \in S : n \leq \tilde{T}^{-1}(\bar{R})\}$ is the set of health deficits for which an individual reaches retirement age; $S = \underline{S} \cup \bar{S}$. If i reaches retirement age and has accumulated $n_2(i)$ health deficits in the retirement period, life-time is given by $\max(\bar{R}, \tilde{T}(n_2(i)))$. As will become apparent, the expected number of health deficits in the retirement period conditional of having acquired $n_1(i)$ deficits in the working period exceeds $n_1(i)$. In sum,¹¹

$$T(i) = \begin{cases} \tilde{T}(n_1(i)) & \text{if } n_1(i) \in \bar{S}, \\ \bar{R} & \text{if } n_1(i) \in \underline{S} \text{ and } n_2(i) \in \bar{S}, \\ \tilde{T}(n_2(i)) & \text{otherwise.} \end{cases} \quad (2)$$

Life-time is finite even without any health deficits during retirement. The healthiest retiree dies at age $T_{\max} \equiv \tilde{T}(0) < \infty$. The individual length of the working period, $R(i)$, is given by

$$R(i) = \tilde{R}(n_1(i), \bar{R}) \equiv \begin{cases} \tilde{T}(n_1(i)) & \text{if } n_1(i) \in \bar{S}, \\ \bar{R} & \text{if } n_1(i) \in \underline{S}. \end{cases} \quad (3)$$

Individuals derive utility from material consumption and disutility from labor supply and the length of the working period. Life-time utility of an individual i reads as

$$U(i) = \int_0^{T(i)} e^{-\rho t} \left(\frac{c(i, t)^{1-\sigma} - 1}{1-\sigma} - \kappa(n_1(i)) \frac{l(i, t)^{1+1/\eta}}{1+1/\eta} \right) dt - V(R(i), n_1(i)), \quad (4)$$

¹¹Our two-period set up in continuous time may imply that an individual surviving to retirement age, \bar{R} , may experience a health shock and immediately die after reaching \bar{R} . In the numerical analysis, reasonably, the mass of individuals dying exactly at age \bar{R} will be negligible.

where t indexes calendar time, $c(i, t)$ and $l(i, t)$ are consumption and labor supply of individual i at time t , respectively, $\rho \in (0, 1]$ is the discount rate, $\sigma > 0$ is the degree of relative risk aversion, and $\eta > 0$ is the Frisch elasticity of labor supply (at the intensive margin). Function $\kappa(n)$ is non-decreasing and captures that a decline in health status may raise the disutility from supplying labor at the intensive margin. V represents the disutility from working along the extensive margin, also possibly depending on health deficits at working age. $V(R, n_1)$ is increasing and convex as a function of the length of the working period, R , and non-decreasing in the number of health deficits during the working period, n_1 . Also suppose that V has weakly increasing differences, i.e., if anything, a marginal increase in the length of the working period has a larger impact on the disutility of work when the worker is less healthy; formally, we assume that $V(R, n'_1) - V(R, n_1)$ is non-decreasing in R whenever $n'_1 > n_1$.

According to (2)–(4), health deficits affect utility only via reducing life expectancy, not by reducing the marginal utility from consumption (as in Becker, 2007, but unlike suggested by Finkelstein et al., 2013). We deliberately focus on this case to obtain a conservative value for the welfare-maximizing level of health spending. If the welfare analysis shows that the status quo of health spending is not too high, then there is a strong argument for maintaining the level of health spending and possibly for a drastic increase of health spending as a response to improvements in medical technology. Another important feature is that the length of life enters “linearly” into life-time utility, whereas the marginal utility of consumption within a period is decreasing.¹²

We focus the analysis on a steady state where the composition of cohorts is the same at each point in time. Each individual possesses the same amount of financial assets during working age, \mathbf{a} .¹³ We impose the standard assumption that the interest rate equals the

¹²We focus on standard preferences. Our results may change after the introduction of unconventional preferences, like non-additive preferences, hyperbolic discounting, consumption habits, and adaptation to poor health. Bommier (2013), for example, introduces “multiplicative preferences” in a life-cycle framework with mortality risk, which induces risk aversion with respect to life length. In our framework risk aversion of this type presumably strengthens the case for raising health expenditure.

¹³This assumption is made in order to improve the calibration of calibration, in particular with respect to the Frisch elasticity of labor supply, η . We implicitly assume that financial wealth is passed on from parents to children when entering retirement. Working aged individuals reach the retirement age with high probability and the size of the working aged population remains approximately constant also when considering health policy changes in our analysis.

discount rate, $r = \rho$. Since individuals rely on the pension system for old age consumption, they are perfectly smoothing consumption during the working period, i.e., for all $t \in [0, R(i)]$,

$$c(i, t) = (1 - \tau_w - \tau_h - \tau_s)wl(i, t) + y, \quad (5)$$

where $y = r\mathbf{a} + \mathcal{T} \equiv \tilde{y}(\mathcal{T})$. The first-order condition on labor supply implies that at each instant the marginal rate of substitution between consumption and labor supply equals the net wage rate. Hence, using (5), $y = \tilde{y}(\mathcal{T})$, and denoting $\tilde{w}(\boldsymbol{\tau}) \equiv (1 - \tau_w - \tau_h - \tau_s)w$, $\boldsymbol{\tau} \equiv (\tau_h, \tau_s, \tau_w)$, labor supply of individual i is implicitly given by the condition

$$\kappa(n_1(i))l(i, t)^{1/\eta} [\tilde{w}(\boldsymbol{\tau})l(i, t) + \tilde{y}(\mathcal{T})]^\sigma = \tilde{w}(\boldsymbol{\tau}). \quad (6)$$

For all $t \in [0, R(i)]$, individual labor supply can thus be expressed as a function of health deficits at working age, n_1 , the net wage rate \tilde{w} , and non-labor income \tilde{y} :

$$l(i, t) = \tilde{l}(n_1(i), \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\mathcal{T})). \quad (7)$$

Thus, if $\kappa' > 0$, labor supply is lower for individuals with more health deficits. The case where $\partial \tilde{l}(n_1, \tilde{w}, y)/\partial n_1 < 0$ is consistent with evidence provided by Cai et al. (2014), showing that individuals who experience moderate health shocks respond by incremental reductions in labor supply. Labor supply is decreasing in non-labor income, $\partial \tilde{l}(n_1, \tilde{w}, y)/\partial y < 0$, a standard property. We will calibrate the model such that also $\partial \tilde{l}(n_1, \tilde{w}, y)/\partial \tilde{w} > 0$ holds, i.e. labor supply is strictly decreasing in social insurance contribution rates τ_h and τ_s .

According to (5) and (7), consumption of individual i during the working period reads, for all $t \in [0, R(i)]$, as

$$c(i, t) = \tilde{w}(\boldsymbol{\tau})\tilde{l}(n_1(i), \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\mathcal{T})) + \tilde{y}(\mathcal{T}) \equiv \tilde{C}_1(n_1(i), \boldsymbol{\tau}, \mathcal{T}). \quad (8)$$

2.4 Distribution and Evolution of Health Deficits

The empirical evidence (e.g. Mitnitski et al., 2006, 2007a, 2007b) suggests that the number of health deficits is Poisson-distributed. Let

$$g(n_j, \lambda_j) = e^{-\lambda_j} \frac{(\lambda_j)^{n_j}}{n_j!} \quad (9)$$

denote the probability density function (p.d.f.) of health deficits in period $j \in \{1, 2\}$ of life. The Poisson parameters λ_1 and λ_2 (the average number and variance of health deficits in period 1 and 2, respectively) depend on productivity-adjusted per capita health spending levels (measured in terms of the numeraire).¹⁴ We thereby take a macroeconomic perspective on the relation between health expenditure and the distribution of health deficits in the population.¹⁵ Health spending can be interpreted as being both preventive and curative.

Denoting health spending per capita targeted to the working-aged and retirees by $\mathfrak{h}_1, \mathfrak{h}_2$ and defining productivity-adjusted spending levels $h_1 \equiv \mathfrak{h}_1/A, h_2 \equiv \mathfrak{h}_2/A$ for period 1 and 2, respectively, we specify

$$\lambda_1 = \tilde{a}_1(h_1), \quad (10)$$

$$\lambda_2 = \tilde{a}_2(h_2) + bn_1, \quad (11)$$

where the \tilde{a}_j 's are functions with properties $\tilde{a}'_j < 0$ and $\tilde{a}''_j > 0, j \in \{1, 2\}$. The convexity assumptions capture the notion that the negative effect of higher health expenditure on health deficits is strictly decreasing. $b > 0$ is a parameter that is independent of health

¹⁴For instance, suppose the health sector employs labor as input and wage costs rise proportionally (and recall that the wage rate w is proportional to A). In this case, to maintain the amount of health services after an increase in total factor productivity, A , health spending has to increase proportionally with A .

¹⁵Underlying this relation is a microeconomic allocation of health expenditure to individuals, which we do not model explicitly beyond the technology specification in the calibration of the model. Our calibration strategy involves matching aggregate health inputs and the distribution of health deficits in the population of working-aged and retirees. Examples of health spending targeted to the working-aged would be expenses for mass examinations of the health status of pupils at schools, costs for educational health campaigns (about nutrition, usage of soft drugs, prevention of HIV infections etc.), and expenses for treating typical health problems among younger adults, like type 1 diabetes, virus infections, bacterial infections, orthopedic issues after accidents and psychiatric problems. Examples of expenses affecting the distribution of health deficits of retirees conditional on the distribution of health deficits of the working-aged are those treating cardiovascular diseases, type 2 diabetes, cancer, stroke, lung disease, and arthritis.

spending. It captures that the number of health deficits in retirement age, n_2 , is path-dependent in a stochastic sense. That is, the distribution of n_2 is conditional on n_1 . The path-dependency of health deficits is consistent with evidence from gerontology suggesting that the probability to get another health deficit next period depends positively on the number of already accumulated health deficits. Using (10) and (11) in (9), the joint p.d.f. of (n_1, n_2) is given by

$$G(n_1, n_2, h_1, h_2) \equiv g(n_1, \tilde{a}_1(h_1))g(n_2, \tilde{a}_2(h_2) + bn_1). \quad (12)$$

According to (2) and (9)–(12), life expectancy at birth (LE) is increasing in health spending and reads as

$$\begin{aligned} \text{LE} &= \sum_{n_1 \in \bar{S}} g(n_1, \tilde{a}_1(h_1))\tilde{T}(n_1) + \bar{R} \sum_{n_1 \in \underline{S}} \sum_{n_2 \in \bar{S}} G(n_1, n_2, h_1, h_2) + \\ &\sum_{n_1 \in \underline{S}} \sum_{n_2 \in \underline{S}} G(n_1, n_2, h_1, h_2)\tilde{T}(n_2) \equiv \widetilde{\text{LE}}(h_1, h_2). \end{aligned} \quad (13)$$

Denote by N_1 and N_2 the size of the population in working age and the number of retirees, respectively. Summing the survivors in working age over all cohorts, the number of workers reads as

$$N_1 = \sum_{n_1 \in \underline{S}} g(n_1, \tilde{a}_1(h_1))\tilde{R}(n_1, \bar{R}) \equiv \tilde{N}_1(h_1, \bar{R}). \quad (14)$$

It is easy to see that \tilde{N}_1 is non-decreasing in h_1 and increasing in \bar{R} . If all individuals reach the retirement age ($\bar{S} = \emptyset$), then $N_1 = \bar{R}$. Using (12), the number of retirees, N_2 , can be written as

$$N_2 = \sum_{n_1 \in \underline{S}} \sum_{n_2 \in \underline{S}} G(n_1, n_2, h_1, h_2) \left(\tilde{T}(n_2) - \bar{R} \right) \equiv \tilde{N}_2(h_1, h_2, \bar{R}). \quad (15)$$

Because lowering the number of health deficits raises life-time and because health deficits are path-dependent, \tilde{N}_2 is increasing in both h_1 and h_2 . Moreover, \tilde{N}_2 is decreasing in \bar{R} .

2.5 Government Budget Constraints

The government budget constraints reflect the macroeconomic trade-offs faced by the social planner and are derived next. For tractability of the complex framework with two periods of endogenous and stochastic lengths, we focus on long-run equilibria (steady states).

2.5.1 Pension Payment Constraint

For pension payments, consider first the properties of the “dependency ratio”, defined as the number of beneficiaries per worker,

$$D = \frac{N_2}{N_1} = \frac{\tilde{N}_2(h_1, h_2, \bar{R})}{\tilde{N}_1(h_1, \bar{R})} \equiv \tilde{D}(h_1, h_2, \bar{R}). \quad (16)$$

Lemma 1. *The dependency ratio function, \tilde{D} , is increasing in health spending targeted to the elderly, h_2 , and decreasing in the statutory retirement age, \bar{R} . The impact of an increase in h_1 on \tilde{D} is generally ambiguous; it is positive if all individuals reach the retirement age ($\bar{S} = \emptyset$).*

Proof. Follows from (16) in view of the properties $\partial \tilde{N}_1 / \partial h_1 \geq 0$ (with equality if $\bar{S} = \emptyset$), $\partial \tilde{N}_1 / \partial \bar{R} > 0$, $\partial \tilde{N}_2 / \partial \bar{R} < 0$, $\partial \tilde{N}_2 / \partial h_1 > 0$, and $\partial \tilde{N}_2 / \partial h_2 > 0$. ■

Lemma 1 suggests that higher health spending for the elderly has a dismal effect on pension benefits, by raising life expectancy and thus also the dependency ratio. Because of the stochastic path-dependency of health deficits, the same may hold when increasing health spending for the working-aged. The effect is generally ambiguous though because an increase in h_1 may help more people to survive until \bar{R} .

Recall that the government perfectly smoothes consumption during the retirement period by paying out an individual-specific and time-invariant pension income, denoted by $P(i)$ for individual i . Consistent with the “equivalence principle” in the German pension system, we assume that pension benefits are proportional to their contributions at working age.¹⁶ The

¹⁶There are only a few exceptions from the “equivalence principle” in Germany. First, after reaching retirement age, mothers receive some benefits depending on the years they raised children. These benefits are partly financed from other budgets than from pension contributions. Second, in 2016, only 2.6 percent of retirees received additional pension benefits to secure a minimum standard of living (Deutsche Rentenversicherung, 2017, p. 275), following an application that has to meet strictly defined criteria.

ratio of pension benefits of two individuals who reach the retirement period is equal to the ratio of their labor income. Thus, for two individuals i and i' ,

$$\frac{P(i)}{P(i')} = \frac{\tilde{l}(n_1(i), \tilde{w}, y)}{\tilde{l}(n_1(i'), \tilde{w}, y)}. \quad (17)$$

Denote by P^{\max} the pension benefit of a retiree who had no health deficits during the working period and thus supplied $\tilde{l}(0, \tilde{w}, y)$ units of labor. According to (17), for any i we have

$$P(i) = \tilde{l}(n_1(i), \tilde{w}, y) \frac{P^{\max}}{\tilde{l}(0, \tilde{w}, y)}. \quad (18)$$

According to (3) and (7), total labor input is

$$L = \sum_{n_1 \in \mathcal{S}} g(n_1, \tilde{a}_1(h_1)) \tilde{R}(n_1, \bar{R}) \tilde{l}(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\mathcal{T})) \equiv \tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T}). \quad (19)$$

In a PAYG pension system, the total revenue from the pension contributions, $\tau_s w L$, must equal the aggregate expenses. Thus, using (18) and (19),

$$\tau_s w \tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T}) = \frac{P^{\max}}{\tilde{l}(0, \tilde{w}, y)} \sum_{n'_1 \in \underline{\mathcal{S}}} \sum_{n'_2 \in \underline{\mathcal{S}}} G(n'_1, n'_2, h_1, h_2) \left[\tilde{T}(n'_2) - \bar{R} \right] \tilde{l}(n'_1, \tilde{w}, y). \quad (20)$$

Solving (20) for $P^{\max}/\tilde{l}(0, \tilde{w}, y)$, inserting into (18) and using $\lambda_1 = \tilde{a}_1(h_1)$, $y = \tilde{y}(\mathcal{T})$ and $L = \tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T})$ implies that pension income of beneficiary i is given by

$$\begin{aligned} P(i) &= \frac{\tau_s w \tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T}) \tilde{l}(n_1(i))}{\sum_{n'_1 \in \underline{\mathcal{S}}} \sum_{n'_2 \in \underline{\mathcal{S}}} G(n'_1, n'_2, h_1, h_2) \left[\tilde{T}(n'_2) - \bar{R} \right] \tilde{l}(n'_1, \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\mathcal{T}))} \\ &\equiv \tilde{P}(n_1(i), h_1, h_2, \bar{R}, \boldsymbol{\tau}, \mathcal{T}), \quad n_1(i) \in \underline{\mathcal{S}}. \end{aligned} \quad (21)$$

Since retirees have to contribute to the health system as well, consumption of retiree i is, for all $t \in [\bar{R}, T(i)]$, given by

$$c(i, t) = (1 - \tau_h) \tilde{P}(n_1(i), h_1, h_2, \bar{R}, \boldsymbol{\tau}, \mathcal{T}) \equiv C_2(n_1(i), h_1, h_2, \bar{R}, \boldsymbol{\tau}, \mathcal{T}). \quad (22)$$

Proposition 1. *The PAYG pension benefit function \tilde{P} is increasing in the statutory retirement age, \bar{R} , and decreasing in health spending targeted to retirees, h_2 . The effect of an increase in health spending targeted to the working-aged, h_1 , on \tilde{P} is generally ambiguous; it is negative if $\kappa' = 0$ and $\bar{S} = \emptyset$.*

Proof. Follows from (21) in view of $\tilde{a}'_1 < 0$, $\tilde{a}'_2 < 0$, and (3). ■

An increase in the statutory retirement age, \bar{R} , raises pension benefits by decreasing the dependency ratio, all other things being equal. An increase in old-age health care spending, h_2 , raises life expectancy and thus lowers pension benefits per retiree, highlighting the interaction between health spending and pension finance. By contrast, an increase in health care spending for workers, h_1 , may as well boost pension benefits. It raises labor supply if $\kappa' > 0$ and helps that fewer individuals die before they reach retirement age (if $\bar{S} \neq \emptyset$). Both effects increase the contributions to the pension system. However, these positive effects do not necessarily dominate the effect originating from the path-dependency of health deficits: as the average number of health deficits prior to retirement is reduced by raising h_1 , life expectancy at retirement age increases, in turn raising the dependency ratio.

2.5.2 Health Expenditure Constraint

Total health spending is given by $N_1\mathfrak{h}_1 + N_2\mathfrak{h}_2$, whereas total revenue is the sum of total contributions from workers, $\tau_h wL$, and retirees, $\tau_h \cdot (\tau_s wL)$.¹⁷ Using (14), (15), $h_1 = \mathfrak{h}_1/A$, $h_2 = \mathfrak{h}_2/A$, $w = A\omega$ and $L = \tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T})$, we obtain

$$\tilde{N}_1(h_1, \bar{R})h_1 + \tilde{N}_2(h_1, h_2, \bar{R})h_2 = \tau_h(1 + \tau_s)\omega\tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T}). \quad (23)$$

According to (23), there is a non-trivial relationship between health spending for the working-aged and health spending for retirees. First, for given revenue in the health system and given population sizes (N_1, N_2), there is a trade-off between h_1 and h_2 since both kinds of spending are basically financed from labor income. Second, if h_1 rises, the distribution of health deficits in the working-aged population improves. If anything, this has positive effects on total labor supply such that the health budget available per retiree is enlarged. On the

¹⁷Recall that $\tau_s wL$ is total revenue in the pension system which is fully paid out at each instant.

one hand, if $\bar{S} \neq \emptyset$ (i.e. not all individuals reach the retirement age) and h_1 increases, more individuals survive to the retirement period. On the other hand, if health status correlates with labor supply ($\kappa' > 0$), at least some workers supply more labor at each instant. Both effects lead to property $\partial \tilde{L} / \partial h_1 > 0$. Third, however, an increase in h_1 means that the population size of retirees, N_2 , increases via the path dependency of health deficits (if $\bar{S} \neq \emptyset$, also N_1 increases), leaving less health spending per retiree.

In the case where the (net) wage elasticity of labor supply is positive, individuals reduce labor supply in response to a higher pension savings rate, τ_s . Finally, a reasonable policy mix would avoid Laffer effects, such that the health budget would enlarge by an increase in both the health contribution rate, τ_h , and the pension contribution rate, τ_h .

2.5.3 Earned Income Tax Credit

The earned income tax credit, \mathcal{T} , under a balanced budget from the purely redistributive labor income taxation is given by τ_w , i.e. $N_1 \mathcal{T} = \tau_w w L$. Using (14) and (19), \mathcal{T} is implicitly given by

$$\tilde{N}_1(h_1, \bar{R}) \mathcal{T} = \tau_w w \tilde{L}(h_1, \bar{R}, \boldsymbol{\tau}, \mathcal{T}). \quad (24)$$

2.6 The Social Planning Problem

Our goal is to numerically derive the simultaneously optimal choices of health spending variables h_1 , h_2 , the health contribution rate τ_h , the pension contribution rate τ_s , and the statutory retirement age \bar{R} that maximize individual expected utility subject to government budget constraints (23) and (24). Given the basic features of the German system, this constitutes the “constrained optimal” social insurance system behind the veil of ignorance (i.e. before individuals draw from the distribution of health deficits early in life and from the conditional distribution of health deficits later in life). Expected utility can be derived by substituting (3), (7), (8) and (22) into (4) and taking expectations. That is, the social planner takes both individual decisions and public finance restrictions into account.¹⁸

¹⁸For the deterministic case, the social planning problem is formally stated and discussed in Appendix B. In Appendix B, we also show that socially optimal health spending generally does not maximize life expectancy at birth except for a special case.

Table 1: Effects of Independently Set Policy Instruments

| | Life expectancy | Individual net labor income | Pension benefits |
|-------------------|------------------------|------------------------------------|-------------------------|
| Rise in h_1 | + | ambiguous | ambiguous |
| Rise in h_2 | + | – | – |
| Rise in \bar{R} | 0 | 0 | + |
| Rise in τ_s | 0 | – | + |

Table 1 summarizes the effects for the welfare determining outcomes of the policy instruments set independently by the social planner.

- An increase in health spending h_1 affects the distribution of health deficits for the working-aged and, via the path-dependency of health deficits, also the distribution of health deficits for retirees. In sum, life expectancy increases. For those with falling health deficits n_1 , labor supply endogenously increases. Net labor income may nevertheless decrease, on average, because of an adjustment in the health contribution rate, τ_h , to fulfill the health financing constraint. Rising life expectancy may well lead to a decrease in pension benefits, despite higher aggregate labor supply that is positively related to pension contributions.
- By improving the distribution of health deficits for retirees, an increase h_2 raises life expectancy at retirement age and thus unambiguously lowers pension benefits. Whereas labor supply is unaffected, net labor income decreases for all individuals since the health contribution rate, τ_h , has to increase.
- An increase in the statutory retirement age, \bar{R} , extends labor supply at the extensive margin while leaving individual labor income per period unaffected. It therefore raises pension benefits by increasing aggregate pension contributions.
- In absence of a Laffer effect, an increase in the pension contribution rate, τ_s , lowers net income for the working aged to the benefit of retirees.

3 Calibration for Germany

We calibrate our model to Germany, which separately has a public PAYG health system with a common health budget for workers and retirees, a public PAYG pension system with linear contributions, and a progressive labor income tax schedule.

We assume that the technology (1) for producing final output has the Cobb-Douglas form $Y = K^{\varrho}(AL)^{1-\varrho}$, $\varrho \in (0, 1)$. For an exogenous interest rate, r , the wage rate is given by $w = A\omega$ with $\omega = (1 - \varrho)(\varrho/r)^{\varrho/(1-\varrho)}$. For later reference, GDP is inferred as $Y = wL/(1 - \varrho)$. Capital income is calibrated at $r\mathbf{a} = \varrho Y$. We set the typical value $\varrho = 1/3$ for the output elasticity of capital.

We interpret a unit of calendar time in the model as 45 years. Assuming that people start on average working at age 20, the working period lasts 45 years, which until recently has been regarded as the normal earnings history in the German system. In 2012, Germany has implemented a stepwise increase of the statutory retirement age from 65 to 67 (one month per year), coming in full effect for those born 1964 or later. For those cohorts, the pension benefit penalty per month of retiring earlier than 67 is 0.3 percentage points, similarly to the current situation for older cohorts. These “early retirement penalties” disincentivize early retirement and are not included in our analysis.¹⁹ In terms of our model, the current statutory retirement age in Germany is thus captured by $\bar{R} = 1$.

We set the annual real interest rate and discount rate to typical values $r = \rho = 0.02$. Consistent with the construction of the health deficit index in the literature, the maximum number of human health deficits is $\bar{n} = 20$. Our results are independent from the metric of health deficits as long as \bar{n} is high enough.²⁰ According to (2), life span is a function of the accumulated health deficits. We specify $\tilde{T}(n) = T_{max} \cdot \exp(-\chi \cdot n)$, $\chi > 0$, and set the maximum life span to $T_{max} = 1.78$, which corresponds to $20 + 1.78 \cdot 45 = 100$ years.

¹⁹There is also the possibility to work part time during retirement (“Altersteilzeit”). In 2016, only 0.23 million out of the 30.51 million workers with compulsory insurance in the public pension system went for this option (Deutsche Rentenversicherung, 2018, Tab. 2). In 2017, Germany also implemented an incentive scheme to work longer than the statutory retirement age. Our analysis implicitly assumes that there is a large extent of behavioral inertia such that the statutory retirement age still remains the focal point.

²⁰Our results are virtually identical when alternatively setting $\bar{n} = 30$ or $\bar{n} = 40$ (not shown). Interestingly, important statistical relations based on the health deficit index are also independent of the number of potential bodily impairments as long as \bar{n} is high enough; see Rockwood and Mitnitski (2007) and Searle et al. (2008).

Formally, employees and employers both contribute to the social insurance system in Germany. Economically relevant is the tax incidence, however. Consistent with our small open economy assumption (i.e. perfectly elastic labor demand), we assume that all pension and health contributions are born by employees. The current pension savings rate, τ_s , is 18.7 percent according to the share of gross wages deducted for social security (*Gesetzliche Rentenversicherung*). The model assumes that there are no private savings for old age. In the case of Germany this seems to be an acceptable approximation since retired households receive about 80 percent of income from social security (see Börsch-Supan and Schnabel, 1998). The health contribution rate, τ_h , is set to 15.5 percent, which is the fraction of gross labor income paid for the German public health care insurance (*Gesetzliche Krankenversicherung*). According to the OECD (2015, Tab. 3.8), the marginal labor income tax rate in Germany for married couples with two children in the year 2014, evaluated at average income, was 26-28 percent (depending on the number of children and whether it is a one-earner or two-earner family). Without children, it was 19 percent in a two-earner family and 21 percent for single earners. We set $\tau_w = 0.25$ and $\tau_s = 0.187$ according to the current contribution rate in the German public pension system.

In our social insurance context, we expect results to respond sensitively to the curvature of the utility function with respect to consumption, parameterized by σ . To calibrate σ , we follow Chetty (2006) and consider an individual for which the part of net income that systematically varies with labor supply, $\tilde{w}\tilde{l}$, and the part that does not varies with labor supply, y , are proportional, $y = \zeta\tilde{w}\tilde{l}$ with $\zeta > 0$. Using (6), it is easy to show that the uncompensated wage elasticity of labor supply is then constant and reads as

$$\left. \frac{\partial \tilde{l}(n_1, \tilde{w}, y)}{\partial \tilde{w}} \right|_{y=\zeta\tilde{w}\tilde{l}(n_1, \tilde{w}, y)} \cdot \frac{\tilde{w}}{\tilde{l}(n_1, \tilde{w}, y)} = \frac{1 + \zeta - \sigma}{\frac{1+\zeta}{\eta} + \sigma} \equiv \varepsilon. \quad (25)$$

It depends on the Frisch wage elasticity, η , the factor of proportionality, ζ , and the coefficient of relative risk aversion, σ . Expression (25) also shows that ε is positive if and only if σ is sufficiently small, which puts an upper bound on σ (Chetty, 2006); that is, $\varepsilon > 0$ if and only if $\sigma < 1 + \zeta$. Naturally, the labor supply elasticity varies with the concept of the household. According to Bargain, Orsini and Peichl (2014), the uncompensated labor supply elasticity

in Germany in the year 2001 when not distinguishing between the intensive and extensive margin is estimated to be 0.14 for men in couples and 0.31 for women in couples. For singles, it is 0.2 for men and 0.18 for women. Looking alone at the intensive margin, estimates are much lower and are basically zero for men. In the benchmark run we set ε to 0.14, the estimated labor supply elasticity for men in couples. Moreover, we assume log-utility for consumption, i.e. $\sigma = 1$, consistent with evidence by Chetty (2006), Engelhardt and Kumar (2009) and Hartley et al. (2013). We also follow Chetty (2006) and assume $\varsigma = 0.5$, which captures an average labor income share of two thirds. According to (25), $\varepsilon = 0.14$, $\sigma = 1$ and $\varsigma = 0.5$ imply the value $\eta = 0.58$ for the Frisch elasticity of labor supply. We provide sensitivity analysis for labor supply elasticities and the curvature of the utility function with respect to consumption.

We allow for health deficits during working age to affect labor supply by specifying $\kappa(n_1) = \kappa_0 e^{\delta n_1}$, $\kappa_0 > 0$, $\delta \geq 0$. According to (6), we then have

$$-\left. \frac{\partial \tilde{l}(n_1, \tilde{w}, y) / \partial n_1}{\tilde{l}(n_1, \tilde{w}, y)} \right|_{y=\varsigma \tilde{w} \tilde{l}(n_1, \tilde{w}, y)} = \frac{(1 + \varsigma)\delta}{\frac{1+\varsigma}{\eta} + \sigma} \equiv \Xi. \quad (26)$$

This suggests that we can approximate $\tilde{l}(n_1, \cdot) / \tilde{l}(0, \cdot) \approx \exp(-\Xi n_1)$. Although empirical evidence shows that individuals with poorer health status retire earlier (Gustman and Steinmeier, 2014), Cai et al. (2014) argue that individuals (presumably those who are not close to retirement age) typically respond to health shocks by gradually reducing labor supply rather than opting out fully. They present evidence on the effect of health shocks and health status at the intensive and the extensive margin. Quantitatively, the bulk of the response to health shocks is at the intensive margin, in line with our model (which ignores the extensive margin for simplicity, unless workers die before reaching the statutory retirement age). According to their Table 2, both men and women with “fair” health (the fourth out of five categories for health status) supply, on average, about 25 percent less working hours than those with “excellent” health (the highest category). Associating “excellent” health with zero health

deficits and “fair” health with three health deficits suggests $\Xi \approx 0.1$.²¹ With $\varsigma = 0.5$, $\sigma = 1$, $\eta = 0.58$ and $\Xi = 0.1$, (26) implies $\delta = 0.25$.

Mitnitski et al. (2007) have shown that the intergenerational distribution of deficits can be precisely described by a Poisson process, as captured by (9). The Poisson parameters λ_1 and λ_2 which determine the arrival of new deficits in the two periods of life, are given by (10) and (11), respectively. We specify

$$\tilde{a}_j(h_j) = \alpha_j \cdot \exp(-\beta_j \cdot h_j), \quad \alpha_j > 0, \quad \beta_j \geq 0, \quad (27)$$

$j \in \{1, 2\}$, to capture in a parsimonious way that the arrival rates for new deficits depend on the general health environment (α_j), and a health technology with decreasing returns of health expenditure (interaction between β_j and h_j). We calibrate the parameters in (27) such that the model approximates actual survival probabilities for each age group. For that purpose we assume that health care expenditure before the 20th century was ineffective in prolonging life of adults (20 years and older), i.e. $\beta_1 = \beta_2 = 0$ for the year 1900 (and earlier). This assumption is approximately true. Before the 20th century life expectancy rose predominantly because of fewer deaths in infancy and childhood. Improving adult life expectancy is a phenomenon of the 20th century. According to Milligan and Wise (2011), mortality at age 65 did not decline substantially until the 1970s. We use the fact that for ages above 20 the force of mortality, that is the conditional probability $\mu(x)$ to die at age x , is precisely measured by *Gompertz law*, $\mu(x) = B \exp(\phi x)$. Using the data from the Human Mortality Database (www.mortality.org), Strulik and Vollmer (2013) estimate $\phi = 0.11$, $B = 0.00001$ for the year 2000 and $\phi = 0.0092$, $B = 0.00078$ for the year 1950. Unfortunately we do not have mortality data for Germany earlier than 1950. Strulik and Vollmer (2013) find that the average Western European values were $\phi = 0.08$, $B = 0.00018$ in 1900. For England and Sweden historical data exists for a longer period. The average European values in the year 1900 are approximately also observed for England in 1850-1900 and for Sweden in 1750-1900 (see Strulik and Vollmer, 2013). The time invariance

²¹For $\Xi = 0.1$, $\tilde{l}(3, \cdot) / \tilde{l}(0, \cdot) = \exp(-0.3) \approx 0.74$. For being consistent with the working hours of those with “poor” health, which are about 75% lower than of those with “excellent” health, when $\Xi = 0.1$, $\tilde{l}(n_1, \cdot) / \tilde{l}(0, \cdot) \approx 0.25$ is reached for $n_1 = 14$.

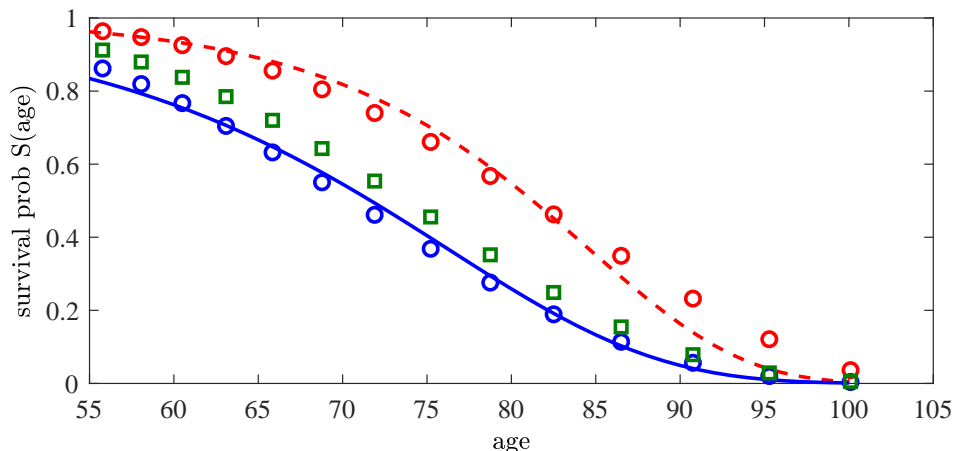
of these numbers is consistent with the general observation that adult mortality was very similar in Western Europe and did not change much before the 20th century. We thus set $\phi = 0.08$ and $B = 0.00018$ for 1900 and earlier and $\phi = 0.11$ and $B = 0.00001$ for the year 2000. From these values we compute the unconditional survival probability $S(x)$ by solving $\dot{S}(x)/S(x) = \mu(x)$ for $S(x)$. The result is shown in Figure 2. The solid blue line shows survival rates in 1900, the red dashed line shows survival rates in 2000.

We begin with estimating χ , α_1 , α_2 , and b such that the predicted age-dependent survival probabilities provide the best fit of the actual survival probabilities in the year 1900 (given $\beta_1 = \beta_2 = 0$). The blue circles in Figure 2 show the implied survival probabilities for $\chi = 0.062$, $\alpha_1 = 1.9$, $\alpha_2 = 3.8$ and $b = 2.5$. How much of the upward shift of the survival curve during the 20th century has been caused by improved health care is a debated issue, which is not yet completely resolved. Much of the improved survival at working age was likely to be driven by improved nutrition and public health measures like sanitation and the implied reduction in the spread of diseases (McKeown, 1976; Fogel; 1994). Old age diseases like cardiovascular diseases and cancer, however, were largely unaffected by these trends and they were actually increasing during the first half of the 20th century. Moreover, the reductions in mortality at old age achieved since the 1950s can be largely attributed to medical innovations and improved medical care (Cutler and Meara, 2001). We take these stylized facts into account and assume for the benchmark run of the model that about 50 percent of improved survival of the working aged is caused by an “improved health environment”, as shown by the green squares in Figure 2. It is reached by an exogenous reduction of α_1 from 1.9 to 1.5, while leaving α_2 unchanged. Notice that survival in retirement improves as well (albeit by less than 50 percent) because of the intergenerational transmission of better health as driven by path-dependency parameter b . We assume that the remainder of the shift of the survival curve has been caused by health technology and health expenditure (as calibrated below).

Disutility of work at the extensive margin (retirement) is driven by a preference for leisure. We specify

$$V(R, n_1) = \nu \cdot (1 + \xi \cdot n_1) \cdot R^{1+1/\gamma}, \quad \nu > 0, \quad \gamma > 0, \quad \xi \geq 0. \quad (28)$$

Figure 2: Actual and Predicted Survival Probability



Survival probabilities in Germany according to Gompertz law in 1900 (solid line) and 2000 (dashed line) and predictions by the model (dots). Squares indicate improvement of survival not originating from improved health care. See text for details.

We set $\gamma = 0.25$, which equals the Frisch elasticity of labor supply at the extensive margin, in line with Chetty et al. (2011). It turns out that the model provides considerable variation in results with respect to alternative assumptions about ξ , i.e. the influence of health on the disutility from work. We set $\xi = 1$ in the benchmark run and provide sensitivity analysis for smaller and larger values. The case $\xi = 1$ means that individuals retire three years later as a response to an improvement of their health deficit index at retirement age from 0.1 (approximately the mode of the distribution of the health deficit index in Germany at age 65) to 0.05 (i.e. a reduction in health deficits from $n_1 = 2$ to $n_1 = 1$ out of $\bar{n} = 20$ potential deficits).²²

This leaves us with four degrees of freedom, the value of ν in the disutility function, the scale parameter A in the production technology, and the health technology parameters β_1 and β_2 . We estimate these parameters such that (i) we approximate the survival curve for the year 2000 (at age 55 and age 80), (ii) the budget constraints of the health and public pension system are fulfilled, (iii) individuals prefer to retire at age 65 and (iv) prefer that a

²²To see this, evaluate the marginal rate of substitution between R and n given utility V , i.e. $V_n/V_R = \xi R(1 + \xi n_1)^{-1}/(1 + 1/\gamma)$, at $\xi = R = 1$, $n_1 = 2$, and $\gamma = 0.25$ (subscripts on V denote partial derivatives). This gives us $V_n/V_R = 1/15$. Recalling that a time period of one corresponds to 45 years in our model, the increase in retirement age which makes the individual indifferent between a reduction of one health deficit and working longer is $45/15 = 3$ years.

working-aged individual has 29 percent of the health expenditure per retiree, $h_1/h_2 = 0.29$, the figure for Germany in the year 2015.²³ This provides the estimates $\nu = 0.163$, $A = 36$, $\beta_1 = 0.81$ and $\beta_2 = 0.68$. The predicted survival curve is shown by red circles in Figure 2. The implied life expectancy at birth, LE as given by (13), is 79.5 years while actually life expectancy at birth was 80.0 years in 2010, according to World Bank, 2015). The predicted GDP share of health care expenditure is 12.2, while actually it was 11.0 percent in the year 2010 (World Bank, 2015).

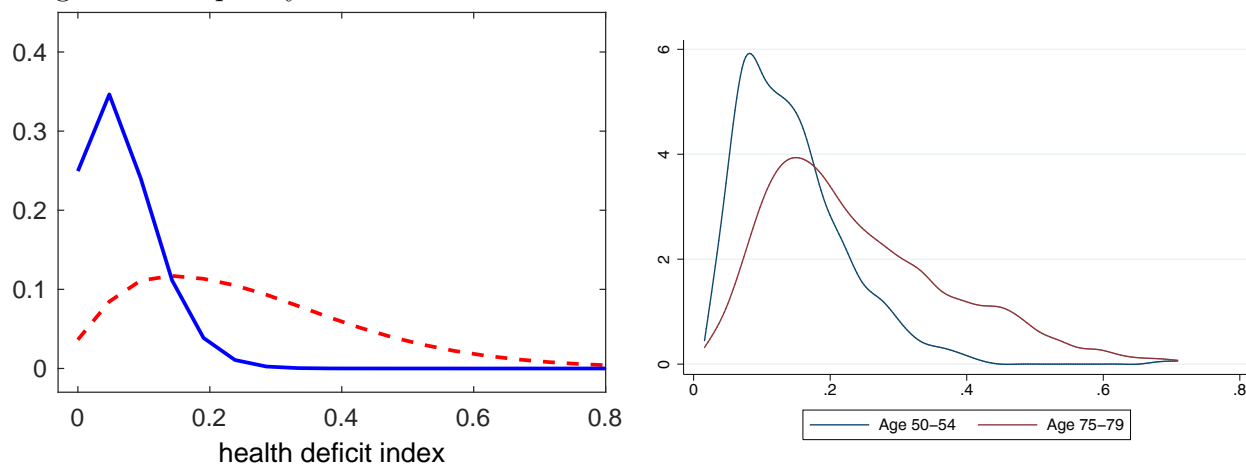
For further comparison with the actual data we compute the implied distribution of the health deficit index, i.e. of the relative number of health deficits out of a long list of potential bodily impairments, conditional on age. Harttgen et al. (2013) have calculated the health deficit index from the ‘Survey of Health, Ageing and Retirement in Europe’ (SHARE) data for several European countries including Germany. Estimates and predictions are shown in Figure 3. Since the maximum number of health deficits in the calibrated model is $\bar{n} = 20$, a health deficit index of, for example, 0.2 means four health deficits. The model approximates the overall distribution reasonably well. The working-aged individuals in our model are a bit too healthy when compared with 50-54 year old persons from the SHARE sample. This seems fine, however, since that cohort is already quite close to the retirement age compared to the average German worker. Unfortunately, SHARE does not provide any data for persons younger than 50. The health deficit distribution of the retired population in our calibrated model corresponds very well to the actual health deficit distribution of the 75-79 year olds.

4 Currently Optimal Social Insurance

In this section we derive the *jointly* optimal social security and health system from an *ex ante* perspective for the current health technology in the calibrated model (future prospects are examined in section 5).

²³We use data from the Statistical Office in Germany on health costs and population sizes split up by age, retrieved on May 10, 2018 (<https://www.destatis.de>). Total health costs for those aged 15-64 and those aged 65+ are about EUR 150 million and 168 million, respectively. The population size of those aged 65+ relative to those aged 15-64 is about 0.32. This implies that health spending per person in the age group 15-64 is about 29 percent of the health spending per person in the age group 65+.

Figure 3: Frequency Distribution of the Health Deficit Index: Calibration vs. Estimation



Left: Predicted frequency distribution of the health deficit index for working age population (solid line) and retired population (dashed line). Right: Estimated frequency distribution of the health deficit index for the age groups 50-54 and 75-79, based on SHARE data in Germany (2007).

4.1 Benchmark Scenario

The first row of Table 2 displays the status quo before optimization. Results for the baseline calibration are shown in the second row, Case 2 (“benchmark”). The best policy is characterized by a decrease of health expenditure as a fraction of labor income from 15.5 to 14.8 percent, i.e. by 4%. The reduction of health care leads to a mild decline of life expectancy at birth, LE, of 0.1 years. Health spending for a younger person should be 24 percent of the health spending for an elderly person, a mild decline compared to the status quo. While the optimal retirement age declines slightly by 0.1 years, the pension contribution rate should increase markedly by 3.2 percentage points. Expected health deficits in working age, $E(n_1)$, and old age, $E(n_2)$, increase marginally but health inequality as measured by Gini coefficients of the health deficit distribution among the young, $Gini_1$, and old, $Gini_2$, remains virtually unchanged at status quo level.

We also computed how much the “value of life” (in monetary terms) changes after a policy reform. The value of life is typically measured as life-time welfare divided by the initial marginal utility of consumption. For simplicity, we refer to the rate of change in life-time welfare W as the percentage change of the value of life, $\Delta W/W$. That is, we evaluate the marginal utility of consumption at the same consumption level before and after the reform.

Going from the status quo to the benchmark case raises the value of life in a negligible fashion, $\Delta W/W = 0.1$ percent (not reported in Table 2). Thus, the benchmark scenario suggests that the current social insurance system in Germany is approximately optimal.

To understand how the previous numerical results depend on the baseline calibration, we now examine the effects of (not necessarily realistic) parameter changes. In order to ensure comparability with the benchmark case we re-calibrated in all subsequent cases the value of ν such retirement at age 65 remains optimal given the new parameter values and the status quo health and pension system.

4.2 The Impact of Health on Disutility from Work

We start with varying ξ , the parameter of which we perhaps know the least. Case 3 investigates the optimal policy when health deficits do not affect the disutility from work at the extensive margin ($\xi = 0$). Naturally, health in working age plays a less important role and the optimal solution is at the corner, $h_1^* = 0$. (Optimal values are denoted by superscript (*)) throughout.) The optimal health contribution rate, τ_h^* , is 2.5 percentage points lower compared to the status quo policy (case 1), reducing life expectancy by 0.3 years. Substituting away from health contributions, individuals prefer to further raise pension contributions as a fraction of labor income, τ_s^* , to 22.4 percent.

Case 4 considers $\xi = 4$, which means that individuals are willing to work four years longer for a reduction of one health deficit. In this case it is optimal to increase the health contribution rate by 1.2 percentage points from the status quo level, shift the health spending structure to the working aged and increase the pension contribution by less than in the benchmark (case 2), to 21.1 percent. Nevertheless, as in Cases 2 and 3, τ_s^* is higher than the current value of 18.7. The optimal retirement age increases by 0.2 year and life expectancy increases by 0.1 year. Thus, a healthy life is from an *ex ante* perspective mildly stronger preferred against high consumption in retirement.

4.3 Labor Supply Elasticities

According to our calibration strategy based on (25), the labor supply elasticities and the elasticity of intertemporal substitution cannot be modified independently. Case 5 in Table 2 shows results for $\eta = 0.3$ (instead of 0.58) and keeping ε at benchmark value. This implies $\sigma = 0.7$, a value close to or below the lower end of empirical estimates. The dominating effect here is the reduced curvature of the utility function, implying that it is now optimal to spend much less on health and consume more during retirement by setting the pension savings rate, τ_s^* , to 22.1 percent. In particular, it is optimal to spend nothing on health at working age and consequently die about 3 years earlier than Case 1 and 2, associated with higher health deficits.

By contrast, in Case 6, a value of $\eta = 0.9$ implies $\sigma = 1.1$, i.e. a relatively minor increase in curvature of the utility function. There is a pronounced impact on preferred health expenditure, in particular in old age. The optimal health contribution rate, τ_h^* , increases mildly by 0.1 percentage points along with a less pronounced increase of the optimal pension savings rate, τ_s^* , to 21.3 percent, compared to the increase in the benchmark case. The optimal retirement age remains at 65 years.

In Case 7 we keep $\eta = 0.58$ from the baseline calibration and set $\varepsilon = 0.05$, a value closer to the estimates of the labor supply elasticity of single men (Bargain et al., 2014). The implied value of σ is 1.3, which causes a significant increase in the curvature of the utility function. As a result it is now optimal to drastically increase health spending such that individuals live for 2.5 years longer than in the status quo case. The optimal retirement age responds also strongly by an increase to 67.3 years. The optimal savings rate, τ_s^* , on the other hand, increases less strongly from the status quo compared to the benchmark case, as a response to the greatly increased health contribution rate and the associated tax distortions of labor supply.

Finally, we investigate with Case 8 the role of pure leisure preference for retirement by increasing γ from 0.25 to 0.5. Consequently, the optimal age of retirement interacts more strongly with health spending, which both rise substantially. Whereas optimal retirement age, \bar{R}^* , is 1.7 years above the status quo, the optimal health contribution rate becomes 18.2

percent. The optimal pension savings rate, $\tau_s^* = 0.193$ is only slightly above the status quo value. The reduction in the expected number of health deficits is associated with an increase in health inequality.

Table 2: Optimal Health and Pension Policy

| Case | | h_1/h_2 | τ_h | τ_s | \bar{R} | LE | $E(n_1)$ | $E(n_2)$ | $Gini_1$ | $Gini_2$ |
|------|-------------------|-----------|----------|----------|-----------|------|----------|----------|----------|----------|
| 1) | status quo | 0.29 | 15.5 | 18.7 | 65.0 | 79.5 | 1.40 | 5.44 | 0.45 | 0.37 |
| 2) | benchmark | 0.24 | 14.8 | 21.9 | 64.9 | 79.4 | 1.41 | 5.47 | 0.45 | 0.37 |
| 3) | $\xi = 0$ | 0.00 | 13.0 | 22.4 | 64.8 | 79.2 | 1.50 | 5.57 | 0.44 | 0.37 |
| 4) | $\xi = 4$ | 0.56 | 16.7 | 21.1 | 65.2 | 79.6 | 1.32 | 5.40 | 0.46 | 0.37 |
| 5) | $\eta = 0.3$ | 0.00 | 5.2 | 22.1 | 64.0 | 76.6 | 1.50 | 6.37 | 0.44 | 0.33 |
| 6) | $\eta = 0.9$ | 0.04 | 15.6 | 21.3 | 65.0 | 79.4 | 1.48 | 5.50 | 0.44 | 0.38 |
| 7) | $\epsilon = 0.05$ | 0.39 | 27.4 | 20.5 | 67.3 | 82.0 | 1.31 | 4.76 | 0.47 | 0.41 |
| 8) | $\gamma = 0.5$ | 0.80 | 18.2 | 19.3 | 66.7 | 80.0 | 1.27 | 5.34 | 0.47 | 0.37 |
| 9) | $\tau_w = 0.3$ | 0.05 | 12.1 | 20.8 | 64.6 | 78.6 | 1.48 | 5.73 | 0.44 | 0.36 |
| 10) | high tech | 2.04 | 24.8 | 17.9 | 67.3 | 82.0 | 1.07 | 4.69 | 0.51 | 0.39 |
| 11) | $\delta = 0$ | 0.09 | 14.4 | 22.5 | 65.0 | 80.0 | 1.46 | 5.30 | 0.45 | 0.39 |

Policy parameters are jointly set to optimal values. \bar{R} is the retirement age converted to years, LE is life expectancy at birth, $E(n_j)$ is expected health deficits in period $j \in \{1, 2\}$, $Gini_j$ is the Gini coefficient for health deficits in period j , τ_h and τ_s are expressed in percent; "high tech" corresponds to the case where all improvement of health for the working-aged from the year 1900 to 2000 can be attributed to advancement in medical technology.

4.4 Other Parameters

Our next numerical experiment considers a higher labor tax rate, $\tau_w = 0.3$ (Case 9). As a consequence, to cope with the high tax distortions of labor supply, τ_h^* and τ_s^* are now both lower than in the benchmark scenario (Case 2). τ_s^* is still above the status quo level though. Per capita health spending is relatively more concentrated on the elderly and the optimal retirement age is slightly lower than in Case 1 and 2.

Case 10 gives us first insights about the role of medical technology. For that purpose we now assume that, albeit unrealistically, all improvement in health during the 20th century can be attributed to medical technology ("high tech" scenario), i.e. we keep α_1 at its level from the year 1900. Fitting the survival curves of Figure 2 requires a re-calibration to $\beta_1 = 1.4$ and $\beta_2 = 1.0$. Not surprisingly it is now optimal to further increase health expenditure, in particular during working age. More health spending implies that the distribution of health

deficits shifts to the left. The Markov-feature of health transitions causes the better health in working age to be transmitted to better health in old age and a longer life. Optimal policy interventions increase the retirement age by 2.3 years and life expectancy by 2.5 years. They substantially reduce the expected number of health deficits for both groups, compared to the status quo. This goes along with an increase in health inequality among both the working aged and the elderly.

Finally, in Case 11 we make the illustrative but empirically refuted assumption that health deficits are irrelevant for labor supply at the intensive margin (by setting $\delta = 0$). Naturally, it is now optimal to shift health expenditure from the young to the elderly. Moreover, total labor supply and pension income are higher than in the benchmark scenario. Thus, life expectancy increases although the health contribution rate declines mildly. The optimal pension contribution rate increases more strongly (compared to benchmark) but the preferred age at retirement remains unchanged.

5 Long-Run Perspectives on Social Insurance, Life Expectancy, and Health Inequality

The analysis in the previous section suggests for realistic parameter values that health spending in Germany is close to optimal, whereas pension savings should be somewhat increased. With respect to health spending, deviations occur in the cases where the curvature in the consumption utility function departs significantly from log-utility or previous advancements in the health technology were very effective in boosting 20th century survival rates. In this section we use the model for out of sample predictions. In particular we are interested in the impact of future advances in medical technology on the optimal social insurance system and on health, life expectancy, and health inequality.

5.1 Optimal Response to Medical Improvements Compared to the Benchmark Run

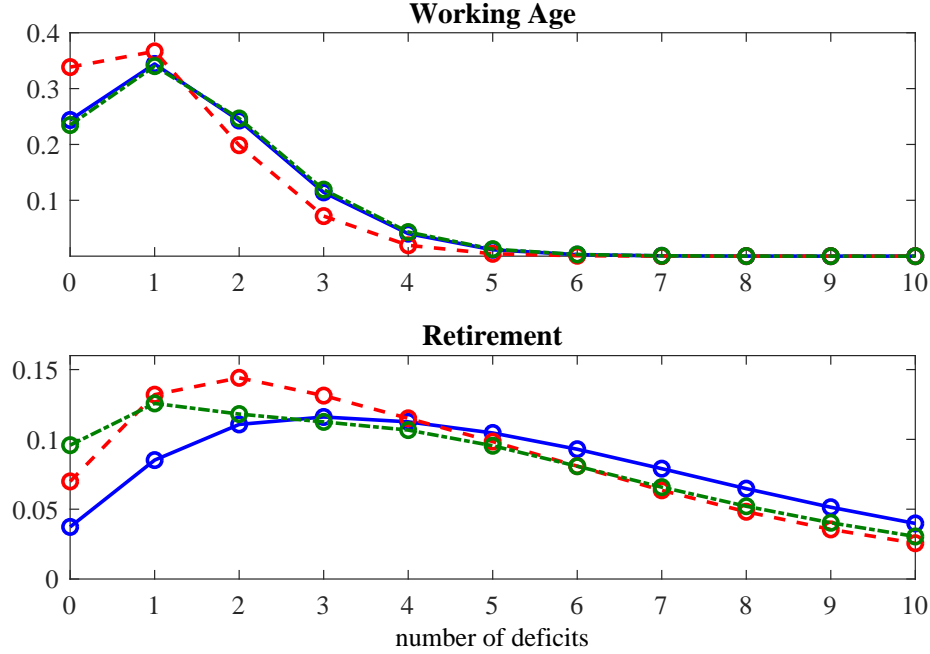
In Case 1 of Table 3, we consider the model as parameterized for the benchmark run except that we increase both technology parameters β_1 and β_2 by factor 1.5. Under the status quo policy mix, this would correspond to an increase in life expectancy from 79.5 to 81.4 years (not shown). Under the optimal adjustment of the social insurance system, life expectancy increases to 83.0 years, another 1.6 years compared to no policy response. This increase comes from higher health expenditure (τ_h^* increases by 5.8 percentage points compared to the benchmark run, Case 2 in Table 2), in particular for individuals at working age whose per capita health spending becomes as high as those of the elderly. Compared to the benchmark, life expectancy extends by 3.6 years, associated with a fall of the expected health deficit index of the old, $E(n_2)/\bar{n}$, by 5.6 percent. Similar to the “high tech” scenario 10 in Table 2, a social planner would raise the retirement age but not by the same extent as life expectancy increases. The policy is accompanied by a savings rate that is 1.7 percentage points higher compared to benchmark, raising the total burden on labor income, $\tau_w + \tau_s^* + \tau_h^*$, by 7.5 percentage points. The larger increase in τ_h^* compared to τ_s^* suggests that, from an *ex ante* perspective, individuals prefer a healthier life against higher consumption in retirement in response to better health technology. The result reflects the fact, emphasized by Hall and Jones (2007), that welfare is linear in the length of life but strictly concave in consumption per period. The last column of Table 3 shows the implied increase in the value of life. The value of life is predicted to substantially increase by $\Delta W/W = 3.4$ percent.

Table 3: Prediction of Future Trends under Optimal Policy Adjustment

| Case | tech. | h_1/h_2 | τ_h | τ_s | \bar{R} | LE | $E(n_1)$ | $E(n_2)$ | $Gini_1$ | $Gini_2$ | $\frac{\Delta W}{W}$ | |
|------|-----------|-----------|----------|----------|-----------|------|----------|----------|----------|----------|----------------------|------|
| 1) | $\xi = 1$ | equal | 1.02 | 20.6 | 20.4 | 66.9 | 83.0 | 1.08 | 4.36 | 0.51 | 0.41 | 3.37 |
| 2) | $\xi = 1$ | biased | 0.13 | 13.7 | 24.6 | 64.8 | 82.3 | 1.45 | 4.61 | 0.45 | 0.43 | 3.68 |
| 3) | $\xi = 0$ | equal | 0.37 | 16.2 | 23.2 | 64.9 | 81.9 | 1.30 | 4.69 | 0.47 | 0.41 | 2.40 |
| 4) | $\xi = 0$ | biased | 0.00 | 12.5 | 24.9 | 64.8 | 82.1 | 1.50 | 4.68 | 0.44 | 0.43 | 3.72 |
| 5) | $\xi = 4$ | equal | 1.50 | 23.1 | 18.6 | 68.2 | 83.5 | 0.98 | 4.24 | 0.53 | 0.41 | 4.03 |
| 6) | $\xi = 4$ | biased | 0.35 | 15.5 | 23.9 | 64.9 | 82.6 | 1.38 | 4.51 | 0.46 | 0.43 | 3.62 |

Other parameters as for benchmark case. tech. equal: β_1 and β_2 increase by 50%. tech. biased: no change of β_1 , β_2 increases by 100%. The last column indicates the welfare gain in percent.

Figure 4: Deficit Distribution: Benchmark Case vs. Long-Run Prediction



Solid lines: optimal actual policy mix. Dashed lines: optimal future policy mix when β_1 and β_2 rise by factor 1.5. Dashed-dotted lines: optimal future policy when β_2 rises by factor 2 and β_1 stays constant.

The most interesting result is perhaps that health inequality is predicted to increase further through medical technological progress accompanied by optimal policy adjustment. $Gini_1$ and $Gini_2$ are given by 0.51 and 0.41, compared to 0.45 and 0.37 in the benchmark run, respectively. Figure 4 illustrates why. Solid lines show the frequency distribution of health deficits for the benchmark case and dashed (red) lines show the predicted distribution. Medical technology significantly improves the health of individuals in anyway good health at the left-hand side of the distribution but it has little impact on the right tail. The share of individuals in full health during working age and those with one or less health deficits in retirement increase substantially.

Case 2 in Table 3 assumes that medical technological progress is age-biased in the sense that all future advancements concern health in old age only (i.e. better treatment of old age diseases). We hold β_1 constant and assume that β_2 doubles. This leads to an increase in life expectancy to 82.3 years with optimal policy response. Compared to the benchmark, the optimal health contribution rate, τ_h^* , declines mildly and health spending is shifted more

to the elderly. The optimal retirement age approximately stays at 65 years and the optimal pension savings rate, τ_s^* , rises by 2.7 percentage points. Lacking technology advancements, health of the young changes only by little compared to the benchmark run. It deteriorates a bit through the shift toward old age expenditure. While expected health deficits of the working-aged, $E(n_1)$, increase somewhat, $Gini_1$ stays constant. Visually, the distribution of health deficits at working age cannot be distinguished from the benchmark case (it lies invisible behind the solid line in the upper panel of Figure 4). Health of the elderly improves significantly. Interestingly, however, $E(n_2)$ is reduced by less than for unbiased technological change, although the elderly experience more medical improvements than in Case 1 of Table 3. The reason is that retirees benefit only directly from technological change but not indirectly through transmission of better health from young age to old age that is at the center of our health deficit approach. The role of health transmission can also be seen by the health deficit distribution in the bottom panel of Figure 4. Dashed-dotted (green) lines visualize the implications of doubling β_2 (and leaving the other parameters unchanged). Due to less transmission of good health there are actually less individuals in retirement with only a few health deficits than under unbiased technological change (dashed red lines).

5.2 Sensitivity Analysis

The remaining cases of Table 3 provide some sensitivity analysis with respect to the role of health deficits in disutility function V . For $\xi = 0$ both unbiased (Case 3) and age-biased (Case 4) medical progress induce a smaller optimal response of health spending than for $\xi = 1$ considered in Case 1 and Case 2, respectively, and consequently life expectancy improves by less relative to the benchmark. The pension savings rate is higher than for $\xi = 1$. With aged-biased progress (Case 4), average health deficits of the young are even higher than in Case 2 due to the extreme shift of health expenditure to the elderly, and health inequality among the young is slightly lower than in Case 2. For $\xi = 4$ (Cases 5 and 6), on the other hand, optimal health spending induced by technological progress should increase more relative to the benchmark than for $\xi = 1$ and should relatively more focus on the young. At the same time, the pension savings rate is lower in order to limit tax distortions of labor

supply. In case of unbiased technological change, this is associated with a significant rise of the desired retirement age, to 68.2 years, reflecting the substantially better health of workers that leads to an increase of life expectancy to 83.5 years. This is also the case that leads to the greatest improvement of the expected value of life and the greatest increase in health inequality among the working-aged.

5.3 Summary and Discussion of Results

Summarizing, our analysis suggests that substantially increasing the health expenditure share can be regarded as optimal when medical technology improves. There are important implications on the distribution of health and the jointly optimal responses of the social insurance system that have yet not been worked out in the literature. While individuals want to exploit the possibility to prolong life by increasing the health contribution rate, they do not necessarily want to raise the pensions savings rate and typically prefer to respond to higher longevity with a higher retirement age. In most cases, individuals prefer to increase the retirement age by a smaller factor than life expectancy, thus re-scaling the life-cycle towards relatively more leisure.

Perhaps surprisingly at first glance, more health spending systematically leads to more health inequality in our calibrated framework. To better understand the generality of this result, recall that the Gini-coefficient is a measure of relative inequality and note that a reduction of health deficits by any number reduces the health deficits of those in good health by relatively more. For example, consider two groups of individuals i and i' with population fraction p and $1 - p$, respectively, and assume that group i has more deficits than i' , $n(i) > n(i')$. Then, if both groups experience a reduction of x health deficits, the healthier group experiences a relatively greater improvement of health. Formally, $\partial [(n - x)/n] / \partial x = -1/n < 0$. Any health intervention x thus improves health of the relatively healthy by relative more and thus increases inequality (according to the Gini coefficient or any other measure of relative inequality). To see this formally, note that before the intervention $x = 0$ and the unhealthy group has the share $pn(i) / [pn(i) + (1 - p)n(i')]$ of all health deficits. Inequality, measured by the Gini coefficient is thus given by $Gini = pn(i) / [pn(i) + (1 - p)n(i')] - p$. After the

intervention, both groups have x health deficits less and the Gini coefficient is

$$Gini = \frac{p \cdot (n(i) - x)}{p \cdot (n(i) - x) + (1 - p) \cdot (n(i') - x)} - p = \frac{1}{1 + \frac{1-p}{p} \frac{n(i')-x}{n(i)-x}} - p \equiv G(x; n(i), n(i')). \quad (29)$$

Thus, we have $\partial G/\partial x > 0$ iff $n(i) > n(i')$, i.e. health inequality increases in the number of reduced health deficits. In sum, technological progress or more health spending, conceptualized as interventions that reduce the (average) number of health deficits, lead to higher relative inequality of health.

6 Conclusion

We integrated into public economics a biologically founded, stochastic process of individual ageing. We derived the optimal design of the social insurance system behind the veil of ignorance for the current health technology and in response to future medical progress. We also investigated the implications for life expectancy and health inequality.

Our results from the calibrated model for Germany suggest that the current social insurance policy instruments are set close to the (constrained) socially optimal levels, given proportional contribution rates for health and pension finance, the equivalence principle in the pension system, and a common statutory retirement age. Interestingly, if anything, the pension savings rate should moderately increase while the desired retirement age is changing very little.

The picture is quite different with further improvements in the medical technology that raise life expectancy, calling for potentially drastic increases in the health contribution rate and little further changes in the optimal pension savings rate. Thus, from an *ex ante* perspective, individuals prefer a healthier and longer life against higher consumption in retirement. The result reflects that life-time utility is linear in the length of life but concave in consumption per period. Typically, the retirement age should increase proportionally less than life expectancy.

Another important insight is that increasing health spending in response to life-prolonging medical innovations raises health inequality. The result is driven by the fact that higher

health spending helps those who are relatively healthy more than those who have accumulated a rather high number of health deficits already. It is interesting in view of the debate on the distribution of health status in the population. For instance, the WHO explicitly aims at reducing “avoidable” health inequity that it is “attributable to the external environment and conditions mainly outside the control of the individuals concerned”. Abstracting from behavioral decisions that may affect individual health, our analysis suggests that this goal may be in conflict with *ex ante* welfare maximization, i.e. behind the veil of ignorance.

In future research we plan to investigate how the incentive to innovate in the pharmaceutical sector interacts with the social insurance system.²⁴ Our framework could also be exploited to examine to which degree health innovations should be promoted vis-à-vis non-health innovations.

Appendix

A. Health Deficit Index and Health Deficit Accumulation

The idea to measure health and aging by a health deficit index (also known as frailty index) has been developed by Mitnitski et al. (2001) and used by hundreds of studies in the medical sciences. The index measures the number of health deficits that a person has at a given age relative to the number of potential health deficits that he or she may have. Health deficits include mild ones as well as serious disabilities. The exact choice of potential deficits is not crucial provided that sufficiently many indicators are present in the index; see Searle et al. (2008) for methodological background. The health deficit index used in Figure 1 has been estimated using the five waves that include health-related information (year 2004 to 2015) from the Survey of Health, Aging and Retirement in Europe (SHARE dataset release 6.0.0). Abeliasky and Strulik (2018) considered the same 38 symptoms, signs and disease classifications, as Harttgen et al. (2013), which are summarized in Table A.1. Multilevel deficits were coded using a mapping to the Likert scale in the interval 0-1.

²⁴Grossmann (2013) studies the role of institutional regulations in the pharmaceutical sector and co-insurance schemes in the health system on pharmaceutical innovations. However, he does not capture the interactions with the social security system and does not endogenize life expectancy.

Table A.1 Items of the Health Deficit Index

| | |
|-----------------------------------|--------------------------------------|
| Arthritis | Difficulties concentrating |
| Stroke | Difficulties shopping |
| Parkinson | Difficulties lifting 5kg |
| Diabetes | Difficulties pulling/pushing object |
| Cholesterol | Less enjoyment |
| Asthma | Difficulties managing money |
| Depression | Difficulties joining activities |
| High blood pressure | Difficulties bathing |
| Cataracts | Difficulties dressing |
| Pain | Difficulties doing housework |
| Difficulties seeing arm length | Difficulties walking across house |
| Difficulties seeing across street | Difficulties eating |
| Difficulties sitting long | Difficulties getting out of bed |
| Difficulties walking 100m | Difficulties using the toilet |
| Difficulties getting out chair | Difficulties using map |
| Difficulties climbing stairs | Walking speed (only in wave 1 and 2) |
| Difficulties kneeling | BMI |
| Difficulties picking an object | Grip strength |
| Difficulties extending arms | Mobility |

Denote the health deficit index of an individual i by D_i . Abeliansky and Strulik (2018) estimated the evolution of health deficits as

$$\ln D_i = \delta_0 + \delta_1 \cdot age_i + \delta_2 \cdot yrbirth_i + \epsilon_i,$$

separately for men and women. Aside from the year-of-birth term, $yrbirth_i$, this regression has been established in gerontology by Mitnitski et al. (2002a, 2002b). The regression equation implies that health deficits grow exponentially with age akin to the Gompertz law of mortality, $D = \delta_0 \cdot \exp(\delta_1 \cdot age)$, in which δ_1 measures the speed of aging (the slope in Figure 1). The size of the parameter δ_2 provides an estimate of the decline of health deficits by year of birth. Figure 1 shows results for Germany obtained by Mundlak-regression. The estimated parameters (and standard errors) for the trajectories displayed in Figure 1 are $\delta_0 = 27.08$ (2.60), $\delta_1 = 0.0187$ (0.0020), and $\delta_2 = -0.0160$ (0.0013) for women and $\delta_0 = 23.85$ (2.88), $\delta_1 = 0.0239$ (0.0028), and $\delta_2 = -0.0139$ (0.0014) for men.

B. Analytic Characterization in the Deterministic Case

Whereas the stochastic model is solved numerically, this appendix analytically derives and discusses the social welfare optimization problem in the deterministic case. Using (3), (7), (8), (22) and $\mathcal{T} \equiv \tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau})$ that is implicitly given by (24) in (4), utility of individual i can be written as

$$U(i) = \begin{cases} \hat{U}(n_1(i), h_1, \bar{R}, \boldsymbol{\tau}) & \text{if } n_1(i) \in \bar{S} \\ \check{U}(n_1(i), h_1, \bar{R}, \boldsymbol{\tau}) & \text{if } n_1(i) \in \underline{S} \wedge n_2(i) \in \bar{S} \\ \tilde{U}(n_1(i), n_2(i), h_1, h_2, \bar{R}, \boldsymbol{\tau}) & \text{otherwise,} \end{cases} \quad (30)$$

where

$$\hat{U}(n_1, h_1, \bar{R}, \boldsymbol{\tau}) \equiv -V(\tilde{T}(n_1), n_1) + \frac{1 - e^{-\rho \tilde{T}(n_1)}}{\rho} \times \left(\frac{\tilde{C}_1(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau}))^{1-\sigma} - 1}{1 - \sigma} - \kappa(n_1) \frac{\tilde{l}(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau})))^{1+1/\eta}}{1 + 1/\eta} \right), \quad (31)$$

$$\check{U}(n_1, h_1, \bar{R}, \boldsymbol{\tau}) \equiv -V(\bar{R}, n_1) + \frac{1 - e^{-\rho \bar{R}}}{\rho} \times \left(\frac{\tilde{C}_1(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau}))^{1-\sigma} - 1}{1 - \sigma} - \kappa(n_1) \frac{\tilde{l}(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau})))^{1+1/\eta}}{1 + 1/\eta} \right), \quad (32)$$

$$\tilde{U}(n_1, n_2, h_1, h_2, \bar{R}, \boldsymbol{\tau}) \equiv -V(\bar{R}, n_1) + \frac{1 - e^{-\rho \bar{R}}}{\rho} \times \left(\frac{\tilde{C}_1(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau}))^{1-\sigma} - 1}{1 - \sigma} - \kappa(n_1) \frac{\tilde{l}(n_1, \tilde{w}(\boldsymbol{\tau}), \tilde{y}(\tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau})))^{1+1/\eta}}{1 + 1/\eta} \right) + \frac{e^{-\rho \bar{R}} - e^{-\rho \tilde{T}(n_2)}}{\rho} \left(\frac{\tilde{C}_2(n_1, h_1, h_2, \bar{R}, \boldsymbol{\tau}, \tilde{\mathcal{T}}(h_1, \bar{R}, \boldsymbol{\tau}))^{1-\sigma} - 1}{1 - \sigma} \right). \quad (33)$$

Expected welfare then reads as

$$\begin{aligned}
W(h_1, h_2, \bar{R}, \boldsymbol{\tau}) \equiv & \sum_{n_1 \in \bar{S}} g(n_1, \tilde{a}_1(h_1)) \hat{U}(n_1, h_1, \bar{R}, \boldsymbol{\tau}) + \sum_{n_1 \in \underline{S}} \sum_{n_2 \in \bar{S}} G(n_1, n_2, h_1, h_2) \check{U}(n_1, h_1, \bar{R}, \boldsymbol{\tau}) + \\
& \sum_{n_1 \in \underline{S}} \sum_{n_2 \in \underline{S}} G(n_1, n_2, h_1, h_2) \tilde{U}(n_1, n_2, h_1, h_2, \bar{R}, \boldsymbol{\tau}). \tag{34}
\end{aligned}$$

Thus, the optimal policy mix solves

$$\max_{h_1, h_2, \tau_h, \tau_s, \bar{R}} W(h_1, h_2, \bar{R}, \boldsymbol{\tau}) \text{ s.t. (23)}, \tag{35}$$

where labor supply functions $\tilde{l}(n_1, \cdot)$, $n_1 \in S$, are implicitly defined by (6).

Because of the various interactions between the health and pension system, (35) is a very complex optimization problem. For instance, consider the welfare interaction of the pension contribution rate, τ_s , with the health contribution rate, τ_h . On the one hand, raising τ_h may make an increase in τ_s less worthwhile and vice versa because contributions to the health system and the pension system come from the same source (labor earnings) and the marginal utility of consumption is declining. On the other hand, an increase in τ_h implies that individuals live longer, all other things equal, thus prolonging the retirement period. This raises the benefit to contribute more to the pension system, i.e. to increase τ_s together with τ_h . If τ_h is increased such that life-time expands, it may seem a good idea to raise retirement age, \bar{R} , as well. This is often suggested in debates on demographic change. However, if \bar{R} increases, the number of contributors to both tiers of the social insurance system, N_1 , rises. It is thus not clear if contribution rates to either form of social insurance should be positively or negatively associated with the retirement age.

The analysis of the numerically calibrated version of the model disambiguates these analytical considerations to derive the socially optimal insurance system and assesses its implications on health inequality and welfare. It is instructive to look first at the simpler, deterministic case, where all individuals are identical also *ex post* and reach the retirement age. Since all individuals are identical, it is meaningless to assume redistribution among workers. Thus, $\tau_w = \mathcal{T} = 0$ and non-labor income y of workers is exogenous.

Define the net wage function for $\tau_w = 0$ as $\hat{w}(\tau_h, \tau_s) \equiv \tilde{w}(\tau_h, \tau_s, 0)$. As $R(i) = \bar{R}$ for all i and cohort size is normalized to unity, the mass of working-aged individuals and retirees is

$$N_1 = \bar{R}, \quad N_2 = \tilde{T}(n_2) - \bar{R}, \quad (36)$$

respectively. In view of (10) and (11), with a degenerated density function g , the number of health deficits of each individual in period 1 and 2 of life equals

$$n_1 = a_1 = \tilde{a}_1(h_1), \quad (37)$$

$$n_2 = a_2 + bn_1 = \tilde{a}_2(h_2) + b\tilde{a}_1(h_1). \quad (38)$$

The health care budget constraint reads as $N_1 h_1 + N_2 h_2 = \tau_h(1 + \tau_s)\omega L$, where total labor supply is $L = \bar{R}\tilde{l}(n_1, \cdot)$. Using (36), (37) and (38), we thus have

$$h_1 + \left(\frac{\tilde{T}(\tilde{a}_2(h_2) + b\tilde{a}_1(h_1))}{\bar{R}} - 1 \right) h_2 = \tau_h(1 + \tau_s)\omega\tilde{l}(\tilde{a}_1(h_1), \hat{w}(\tau_h, \tau_s), y), \quad (39)$$

implicitly defining health spending for retirees, $h_2 \equiv \tilde{h}_2(h_1, \tau_h, \tau_s, \bar{R})$, as a function of the other policy instruments. Using this in (38) leads to

$$n_2 = \tilde{a}_2(\tilde{h}_2(h_1, \tau_h, \tau_s, \bar{R})) + b\tilde{a}_1(h_1) \equiv \tilde{n}_2(h_1, \tau_h, \tau_s, \bar{R}). \quad (40)$$

According to (8) and (37), at each instant, consumption of working-aged individuals is given by

$$C_1 = \hat{w}(\tau_h, \tau_s)\tilde{l}(\tilde{a}_1(h_1), \hat{w}(\tau_h, \tau_s), y) + y. \quad (41)$$

Equating aggregate expenses to aggregate contributions in the pension system, $N_2 B = N_1 \tau_s \omega \tilde{l}(n_1, \hat{w}(\tau_h, \tau_s), y)$, and using (36), (37) and (40), we find that consumption (net income after paying health contributions) per retiree, $C_2 = (1 - \tau_h)B$, reads at each instant as

$$C_2 = \frac{(1 - \tau_h)\bar{R}\tau_s\omega\tilde{l}(\tilde{a}_1(h_1), \hat{w}(\tau_h, \tau_s), y)}{\tilde{T}(\tilde{n}_2(h_1, \tau_h, \tau_s, \bar{R})) - \bar{R}}. \quad (42)$$

Using (37), (41), (42) and $T = \tilde{T}(n_2)$ in (4), individual welfare reads as

$$\begin{aligned}
U = & -V(\bar{R}, \tilde{a}_1(h_1)) + \frac{1 - e^{-\rho\bar{R}}}{\rho} \times \\
& \left(\frac{\left[\frac{\hat{w}(\tau_h, \tau_s) \tilde{l}(\tilde{a}_1(h_1), \hat{w}(\tau_h, \tau_s), y) + y}{1 - \sigma} \right]^{1-\sigma} - 1}{1 - \sigma} - \kappa(\tilde{a}_1(h_1)) \frac{\tilde{l}(\tilde{a}_1(h_1), \hat{w}(\tau), y)^{1+1/\eta}}{1 + 1/\eta} \right) + \\
& \frac{e^{-\rho\bar{R}} - e^{-\rho\tilde{T}(n_2)}}{\rho} \left(\frac{\left[\frac{(1-\tau_h)\bar{R}\tau_s \tilde{l}(\tilde{a}_1(h_1), \hat{w}(\tau_h, \tau_s), y)}{\tilde{T}(n_2) - \bar{R}} \right]^{1-\sigma} - 1}{1 - \sigma} \right) \equiv u(h_1, \tau_h, \tau_s, \bar{R}, n_2). \quad (43)
\end{aligned}$$

A social planner sets policy parameters to solve

$$\max_{h_1 \geq 0, \tau_h \in [0,1], \tau_s \in [0,1], \bar{R} \in [0, T_{\max}], n_2 \in S} u(h_1, \tau_h, \tau_s, \bar{R}, n_2) \text{ s.t. } n_2 = \tilde{n}_2(h_1, \tau_h, \tau_s, \bar{R}). \quad (44)$$

Denote by $(h_1^*, \tau_h^*, \tau_s^*, \bar{R}^*)$ the solution to (44) with respect to the policy variables. The optimal health spending targeted to the retirees is inferred as $h_2^* \equiv \tilde{h}_2(h_1^*, \tau_h^*, \tau_s^*, \bar{R}^*)$. As claimed in the main text, we now show that the optimal allocation of health spending towards working-aged and retired individuals, (h_1^*, h_2^*) , maximizes life-time.²⁵ To avoid only mildly interesting discussions about potential corner solutions, we focus our analysis on interior solutions of (44).

Proposition A.1. *Suppose that $(h_1^*, \tau_h^*, \tau_s^*, \bar{R}^*)$ is an interior maximizer of (44). Then the optimal allocation of health spending across periods of life maximizes life expectancy if and only if $\kappa' = 0$ and V does not depend on n_1 .*

Proof. Define $\mathbf{z} \equiv (h_1, \tau_h, \tau_s, \bar{R})$. We need to establish that at an interior solution to (44), $\mathbf{z}^* \equiv (h_1^*, \tau_h^*, \tau_s^*, \bar{R}^*)$, the following holds: (i) If $\kappa' = 0$ and V does not depend on n_1 , then $\partial \tilde{n}_2(\mathbf{z}^*) / \partial h_1 = 0$; (ii) if $\kappa' > 0$ or $\partial V / \partial n_1 > 0$, then $\partial \tilde{n}_2(\mathbf{z}^*) / \partial h_1 > 0$. To see this, let us define $\hat{u}(\mathbf{z}) \equiv u(\mathbf{z}, \tilde{n}_2(\mathbf{z}))$, $\Phi(\mathbf{z}, n_2) \equiv \partial u(\mathbf{z}, n_2) / \partial n_2$ and $\hat{\Phi}(\mathbf{z}) \equiv \Phi(\mathbf{z}, \tilde{n}_2(\mathbf{z}))$. Using (43),

²⁵For analytical simplicity, we treat health deficits n_1 and n_2 as (non-negative) real numbers rather than as integers.

we obtain the following partial derivatives of $\hat{u}(\mathbf{z})$ with respect to h_1 and τ_h :

$$\begin{aligned} \frac{\partial \hat{u}}{\partial h_1} &= \hat{\Phi} \frac{\partial \tilde{n}_2}{\partial h_1} + \frac{1 - e^{-\rho \bar{R}}}{\rho} \left(\frac{\partial \tilde{l}}{\partial n_1} \underbrace{[(C_1)^{-\sigma} \hat{w} - \kappa \tilde{l}^{1/\eta}]}_{=0, \text{ acc. to (6)}} - \kappa' \frac{\tilde{l}^{1+1/\eta}}{1 + 1/\eta} \right) \tilde{a}'_1 + \\ &\frac{e^{-\rho \bar{R}} - e^{-\rho \tilde{T}(n_2)}}{\rho} (C_2)^{-\sigma} \frac{\bar{R} w \tau_s}{\tilde{T}(n_2) - \bar{R}} \frac{\partial \tilde{l}}{\partial n_1} \tilde{a}'_1 - \frac{\partial V}{\partial n_1} \tilde{a}'_1, \end{aligned} \quad (45)$$

$$\begin{aligned} \frac{\partial \hat{u}}{\partial \tau_h} &= \hat{\Phi} \frac{\partial \tilde{n}_2}{\partial \tau_h} - \frac{1 - e^{-\rho \bar{R}}}{\rho} \frac{\partial \tilde{l}}{\partial \hat{w}} w \underbrace{[(C_1)^{-\sigma} \hat{w} - \kappa \tilde{l}^{1/\eta}]}_{=0, \text{ acc. to (6)}} - \\ &\frac{e^{-\rho \bar{R}} - e^{-\rho \tilde{T}(n_2)}}{\rho} (C_2)^{-\sigma} \frac{\bar{R} \tau_s w \tilde{l}}{\tilde{T}(n_2) - \bar{R}} \left(\frac{\partial \tilde{l}}{\partial \hat{w}} \frac{w}{\tilde{l}} + 1 \right), \end{aligned} \quad (46)$$

where we used the fact $\partial \hat{w} / \partial \tau_h = -w$ in (46). Also note that $(C_1)^{-\sigma} \hat{w} = \kappa \tilde{l}^{1/\eta}$, according to (6). According to (40), we have

$$\frac{\partial \tilde{n}_2}{\partial h_1} = \tilde{a}'_2 \frac{\partial \tilde{h}_2}{\partial h_1} + b \tilde{a}'_1, \quad (47)$$

$$\frac{\partial \tilde{n}_2}{\partial \tau_h} = \frac{\partial \tilde{h}_2}{\partial \tau_h} \tilde{a}'_2, \quad (48)$$

At the optimum, there cannot be Laffer effects, i.e. $\partial \tilde{h}_2 / \partial \tau_h \geq 0$. Thus, (48) and $\tilde{a}'_2 < 0$ imply that

$$\frac{\partial \tilde{n}_2(\mathbf{z}^*)}{\partial \tau_h} \leq 0. \quad (49)$$

Hence, at an interior solution \mathbf{z}^* to (44), where $\partial \hat{u}(\mathbf{z}^*) / \partial \tau_h = 0$, we have

$$\hat{\Phi}(\mathbf{z}^*) < 0, \quad (50)$$

according to (45). Recall that $\partial \tilde{l} / \partial n_1 < (=) 0$ if and only if $\kappa' > (=) 0$. By definition of \mathbf{z}^* , $\partial \hat{u}(\mathbf{z}^*) / \partial h_1 = 0$. The properties thus follow from (45), (50) and $\tilde{a}'_1 < 0$. This concludes the proof. ■

If an increase in health spending targeted to the working-aged has no effect on labor

supply ($\kappa' = 0$) and individuals do not care about health status per se (i.e. V does not depend on n_1), then the social planner wants to maximize the span of life in which individuals earn retirement income. This is achieved by minimizing health deficits of the elderly, $n_2 = \tilde{n}_2(h_1, \tau_h, \tau_s, \bar{R})$. If $\kappa' > 0$, however, an increase in labor supply that results from an increase in health expenditure, h_1 , raises contributions to the pension system. Hence, it is optimal to sacrifice life-time to improve consumption in each point of time for both working-aged individuals and retirees. Also if workers have direct disutility from illness (V is increasing in n_1), the social planner biases the health spending structure towards workers. Proposition A.1 would also hold under a “constrained optimal policy mix” where pension policy (τ_s, \bar{R}) is treated as given.

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